



# CP Violation From Majorana Phases

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# Majorana CP-Violating Phases

The 3x3 **quark** mixing matrix: 1 ~~CP~~ phase

When  $\bar{\nu}_i = \nu_i$  —

The 3x3 **lepton** mixing matrix: 3 ~~CP~~ phases

The 2 extra phases,  $\alpha_1$  and  $\alpha_2$ , are called **Majorana phases**.

Each Majorana phase is associated with a particular  $\nu$  mass eigenstate  $\nu_i$ :

$$U_{\rho i} = U_{\rho i}^0 e^{i \frac{\alpha_i}{2}} ; \text{ all } \rho . \quad U = \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \\ \left[ \begin{array}{ccc} U_{e1}^0 e^{i \frac{\alpha_1}{2}} & U_{e2}^0 e^{i \frac{\alpha_2}{2}} & U_{e3}^0 \\ U_{\mu 1}^0 e^{i \frac{\alpha_1}{2}} & U_{\mu 2}^0 e^{i \frac{\alpha_2}{2}} & U_{\mu 3}^0 \\ U_{\tau 1}^0 e^{i \frac{\alpha_1}{2}} & U_{\tau 2}^0 e^{i \frac{\alpha_2}{2}} & U_{\tau 3}^0 \end{array} \right] \begin{array}{l} e \\ \mu \\ \tau \end{array} \end{array}$$

Bilenky, Hosek, and Petcov; Schechter and Valle, Doi et al.

An **L-conserving** process:

$$\begin{aligned}
 & \text{Amp}[e^-W^+ \rightarrow \nu \rightarrow \mu^-W^+] \\
 & \sim \sum_i \underbrace{\langle \mu^-W^+ | H | \nu_i \rangle}_{U_{\mu i}} \text{Propagator}(\nu_i) \underbrace{\langle \nu_i | H | e^-W^+ \rangle}_{U_{ei}^*} \\
 & \sim \sum_i U_{\mu i} \text{Propagator}(\nu_i) U_{ei}^*
 \end{aligned}$$

An **L-nonconserving** process:

$$\begin{aligned}
 & \text{Amp}[e^+W^- \rightarrow \nu \rightarrow \mu^-W^+] \\
 & \sim \sum_i \langle \mu^-W^+ | H | \nu_i \rangle \text{Propagator}(\nu_i) \langle \nu_i | H | e^+W^- \rangle \\
 & \text{CTP: } \langle \nu_i | H | e^+W^- \rangle = \langle \nu_i | H | e^-W^+ \rangle^* = U_{ei} \\
 & \text{So Amp}[ \mathcal{L} ] \sim \sum_i U_{\mu i} \text{Propagator}(\nu_i) U_{ei}
 \end{aligned}$$

This is sensitive to Majorana phases.

Majorana phases have physical consequences,  
but only in physical processes that involve  
violation of L.

They do not affect  $\nu$  flavor oscillation, but they  
do affect  $0\nu\beta\beta$ :

$$m_{\beta\beta} = \left| \sum_i m_i U_{ei}^2 \right|$$

clearly depends on the relative phase  
of  $U_{e1}^2$  and  $U_{e2}^2$ .

But the rate for  $0\nu\beta\beta$ , whatever it may be, is not a  
CP-violating difference between a process  
and its CP-mirror image.

# Can Majorana Phases Lead to Manifest ~~CP~~?

Manifest ~~CP~~:

$$\text{Rate [Process]} \neq \text{Rate } [\overline{\text{Process}}]$$

The “Dirac” ~~CP~~ phase in the quark mixing matrix causes such inequalities.

Can Majorana phases cause them too?

Yes, they can cause them in heavy neutrino decay in the early universe (Leptogenesis).

(Fukugita & Yanagida)

But can they cause rate inequalities in present-day processes?

Yes, although only in processes that are extremely difficult to observe.

(de Gouvêa, BK, Mohapatra)

An example is  $e^\pm W^\mp \rightarrow \nu \rightarrow \mu^\mp W^\pm$ .

$$\text{Amp} [e^+W^- \rightarrow \nu \rightarrow \mu^-W^+] =$$

$$= S \sum_i \underbrace{U_{ei} U_{\mu i}}_{\text{Has Maj. phases}} m_i/E \underbrace{\exp\{-im_i^2 (L/2E)\}}_{\text{Distance}} \underbrace{\phantom{\exp\{-im_i^2 (L/2E)\}}}_{\text{v propagator}}$$

Helicity suppression

$$\text{Amp} [e^-W^+ \rightarrow \nu \rightarrow \mu^+W^-] =$$

$$= S \sum_i U_{ei}^* U_{\mu i}^* m_i/E \exp\{-im_i^2 (L/2E)\}$$

Suppose only 2 generations matter:

$$U = \begin{matrix} & \nu_1 & \nu_2 \\ \nu_e & \begin{bmatrix} i\frac{\alpha}{2} & \\ ce & s \end{bmatrix} \\ \nu_\mu & \begin{bmatrix} i\frac{\alpha}{2} & \\ -se & c \end{bmatrix} \end{matrix} \quad \begin{aligned} c &\equiv \cos \theta \\ s &\equiv \sin \theta \\ \alpha &\equiv \text{a Majorana phase.} \end{aligned}$$

$$\Gamma[e^+W^- \rightarrow \nu \rightarrow \mu^-W^+] = K \frac{\sin^2 2\theta}{4E^2} \left[ m_1^2 + m_2^2 - 2m_1 m_2 \cos\left( \Delta m^2 \frac{L}{2E} \ominus \alpha \right) \right]$$

(Schechter & Valle)

$$\Gamma[e^-W^+ \rightarrow \nu \rightarrow \mu^+W^-] = K \frac{\sin^2 2\theta}{4E^2} \left[ m_1^2 + m_2^2 - 2m_1 m_2 \cos\left( \Delta m^2 \frac{L}{2E} \oplus \alpha \right) \right]$$

Here,

$K = \text{irrelevant constant} = |S|^2$

$m_{1,2} = \text{masses of } \nu_{1,2}$

$\Delta m^2 = m_2^2 - m_1^2$

Note the two rates are not the same.

# Why Are There 3 Generations?

If the preponderance of **MATTER** over *antimatter* in the universe arose from  $\cancel{CP}$  in quark mixing, we could argue that—

- It takes  $\geq 3$  generations to have  $\cancel{CP}$  in quark mixing.
- It takes  $\cancel{CP}$  in quark mixing to have **MATTER**  $\gg$  *antimatter*.
- It takes **MATTER**  $\gg$  *antimatter* to have us.

But  $\cancel{CP}$  in quark mixing is completely inadequate for **MATTER**  $\gg$  *antimatter*.

Majorana phases can produce the manifest ~~CP~~

$$\Gamma[\text{N} \rightarrow \ell^+ + \text{Higgs}^-] \neq \Gamma[\text{N} \rightarrow \ell^- + \text{Higgs}^+]$$

in the early universe. *This (Leptogenesis)* may be the origin of **MATTER** >> antimatter.

It takes only **2** generations to have manifest ~~CP~~ from Majorana phases.

So why are there **3??**

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