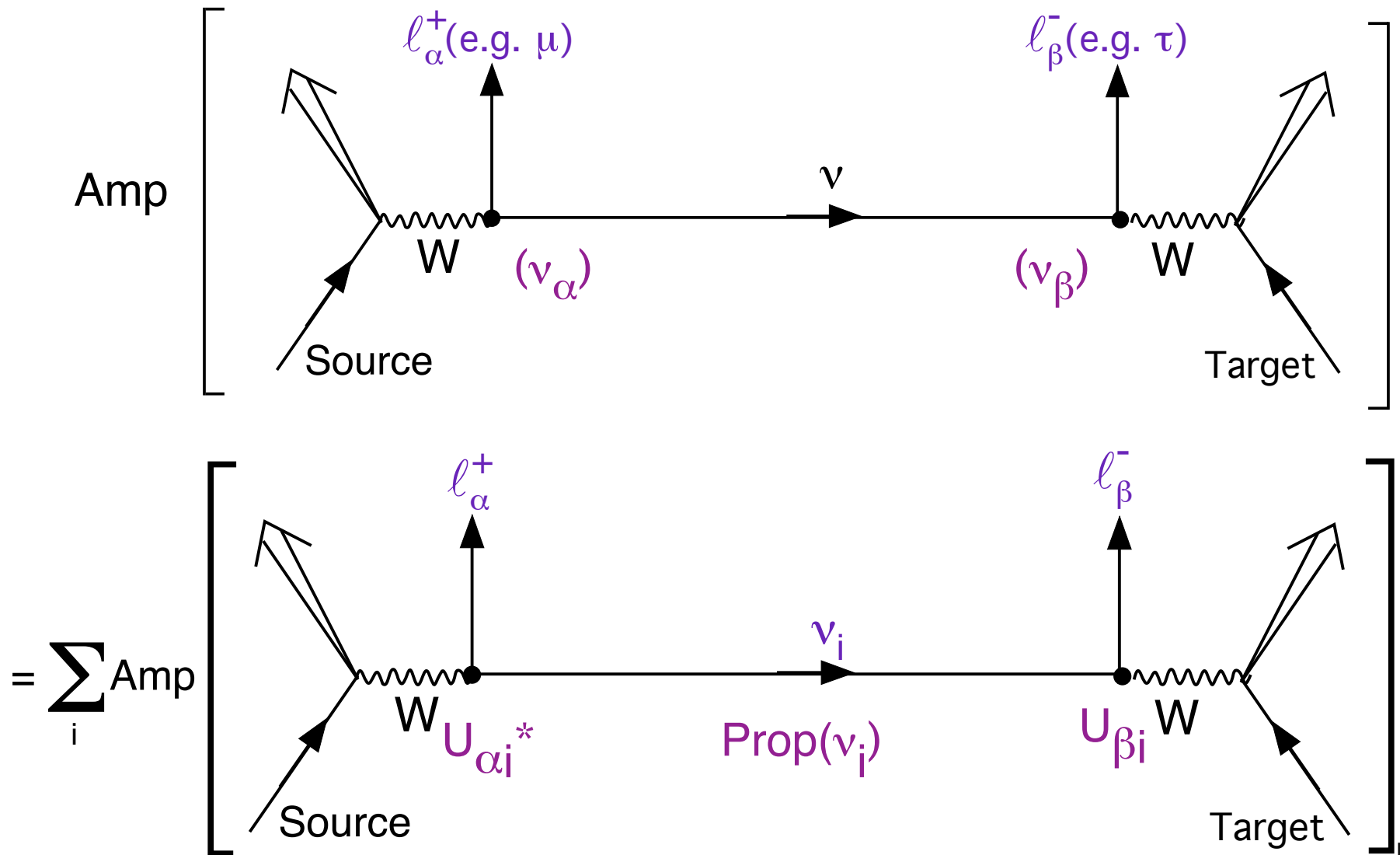


Neutrino Flavor Change (Oscillation) in Vacuum

(Approach of
B.K. & Stodolsky)



$$\text{Amp } [\nu_\alpha \rightarrow \nu_\beta] = \sum U_{\alpha i}^* \text{Prop}(\nu_i) U_{\beta i}$$

What is Propagator $(\nu_i) \equiv \text{Prop}(\nu_i)$?

In the ν_i rest frame, where the proper time is τ_i ,

$$i \frac{\partial}{\partial \tau_i} |\nu_i(\tau_i)\rangle = m_i |\nu_i(\tau_i)\rangle .$$

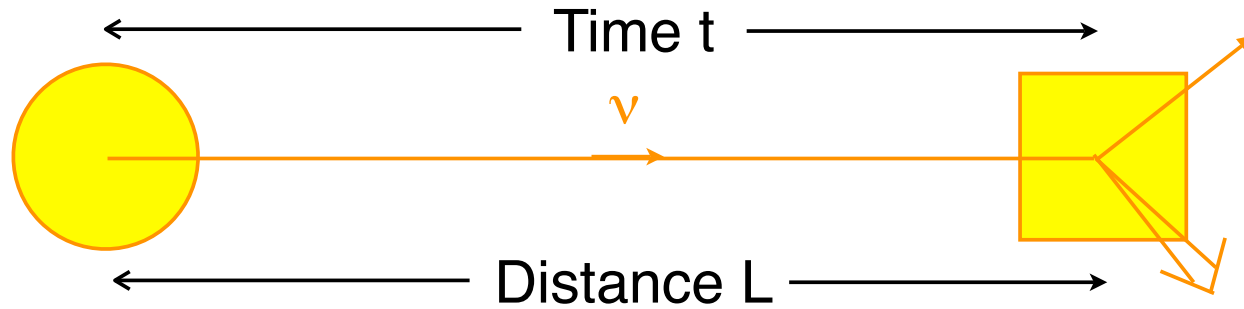
Thus,

$$|\nu_i(\tau_i)\rangle = e^{-im_i\tau_i} |\nu_i(0)\rangle .$$

Then, the amplitude for propagation for time τ_i is —

$$\text{Prop}(\nu_i) \equiv \langle \nu_i(0) | \nu_i(\tau_i) \rangle = e^{-im_i\tau_i} .$$

In the laboratory frame —



The experimenter chooses L and t .

They are common to all components of the beam.

For each v_i , by Lorentz invariance,

$$m_i \tau_i = E_i t - p_i L .$$

Neutrino sources are \sim constant in time.

Averaged over time, the

$$e^{-iE_1 t} - e^{-iE_2 t} \quad \text{interference}$$

is —

$$\langle e^{-i(E_1 - E_2)t} \rangle_t = 0$$

$$\text{unless } E_2 = E_1 .$$

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} .$$

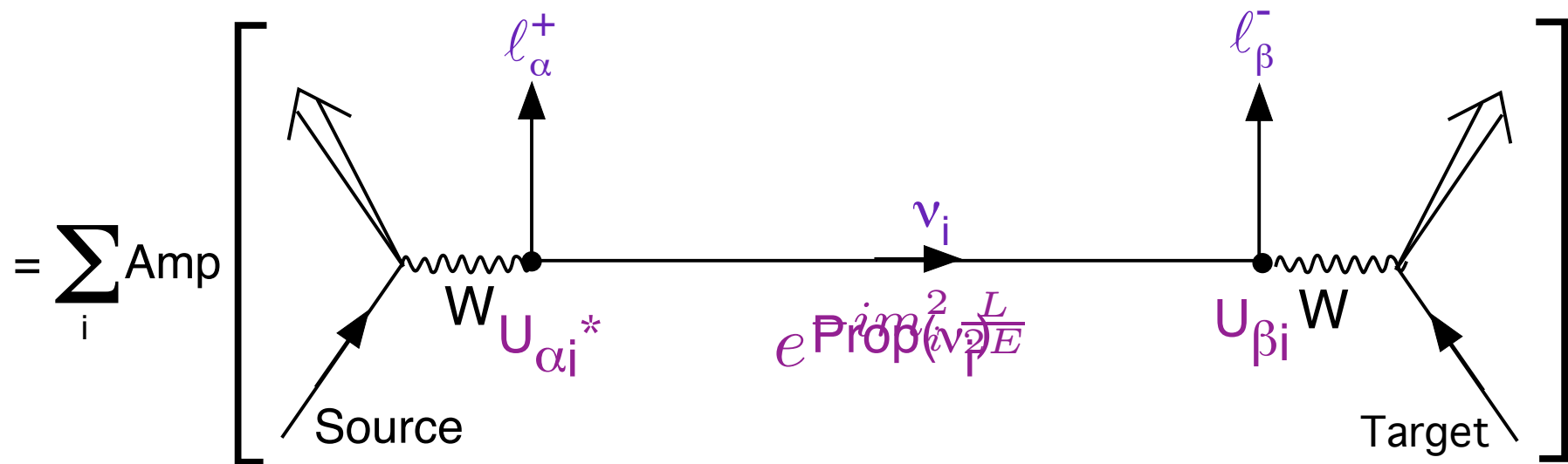
Then the phase in the ν_i propagator $\exp[-im_i\tau_i]$ is —

$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= E(t - L) + m_i^2 L/2E .$$

Irrelevant overall phase \uparrow

Amp $[\nu_\alpha \rightarrow \nu_\beta]$



$$= \sum_i U_{\alpha i}^* e^{-i m_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 = \\ &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ &\quad + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \stackrel{CPT}{=} P(\nu_\beta \rightarrow \nu_\alpha) = P(\nu_\alpha \rightarrow \nu_\beta; U \rightarrow U^*)$$

Thus,

$$\begin{aligned} P(\bar{\nu}_\alpha^{(-)} \rightarrow \bar{\nu}_\beta^{(-)}) &= \\ &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ &\quad \pm 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

A complex U would lead to the CP violation

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta) \quad .$$

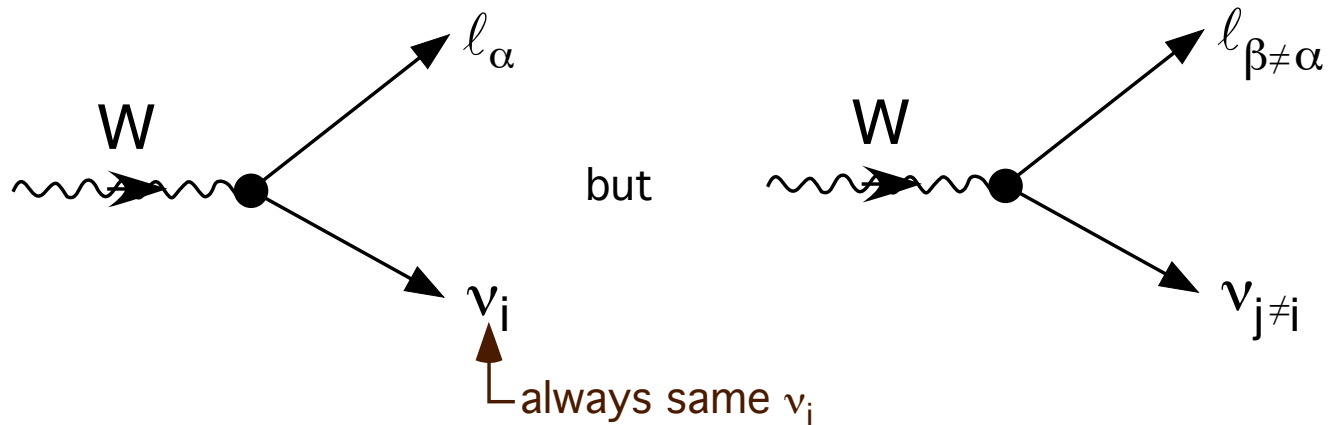
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* \Rightarrow ν Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta \neq \alpha, i} = 0, \text{ so that } P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \delta_{\alpha\beta}.$$

Flavor *change* \Rightarrow Mixing

3. One can detect ($\nu_\alpha \rightarrow \nu_\beta$) in two ways:

See $\nu_{\beta \neq \alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including \hbar and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})}$$

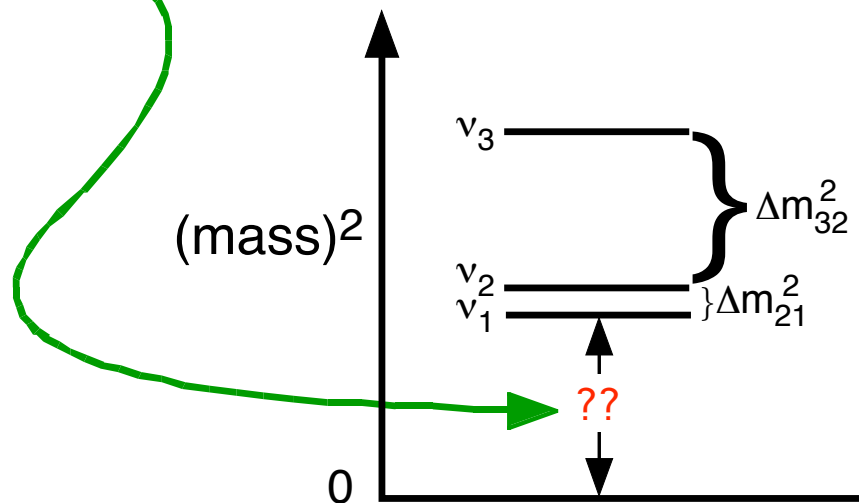
$\sin^2 \left[1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})} \right]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2 (\text{eV}^2) \gtrsim \frac{E(\text{GeV})}{L(\text{km})} .$$

5. Flavor change in vacuum oscillates with L/E . Hence the name “neutrino oscillation”. {The L/E is from the proper time τ .}

6. $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}) = 1$$

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_{\mu}} + \phi_{\nu_{\tau}} < \phi_{\text{Original}}$$

8. Assuming all coherent v_i in a beam have a common **momentum p** , rather than a common energy E , is a harmless error.

This assumption leads to the same $P(\overset{(-)}{v}_\alpha \rightarrow \overset{(-)}{v}_\beta)$.

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$\begin{aligned} e^{-im_1^2 \frac{L}{2E}} \text{Amp}^*(\nu_\alpha \rightarrow \nu_\beta) &= \sum_i U_{\alpha i} U_{\beta i}^* e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}} \\ &= U_{\alpha 3} U_{\beta 3}^* e^{2i\Delta_{31}} + U_{\alpha 2} U_{\beta 2}^* e^{2i\Delta_{21}} + \underbrace{(U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^*)}_{\text{Unitarity}} \\ &= 2i[U_{\alpha 3} U_{\beta 3}^* e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U_{\beta 2}^* e^{i\Delta_{21}} \sin \Delta_{21}] \end{aligned}$$

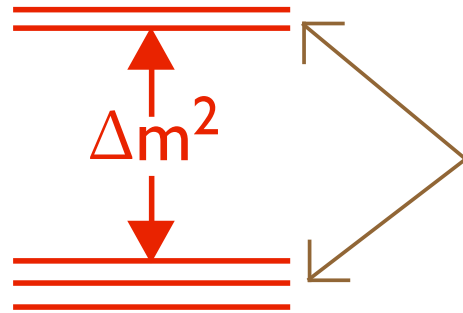
$$\text{where } \Delta_{ij} \equiv \Delta m_{ij}^2 \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} .$$

$$\begin{aligned}
P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) &= \left| e^{-im_1^2 \frac{L}{2E}} \text{Amp}^* (\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) \right|^2 \\
&= 4[|U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2} U_{\beta 2}|^2 \sin^2 \Delta_{21} \\
&\quad + 2|U_{\alpha 3} U_{\beta 3} U_{\alpha 2} U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} (\pm) \delta_{32})] .
\end{aligned}$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3} U_{\beta 3}^* U_{\alpha 2}^* U_{\beta 2})$, a CP – violating phase.

Two waves of different frequencies,
and their ~~CP~~ interference.

When One Big Δm^2 Dominates



These splittings are invisible if $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

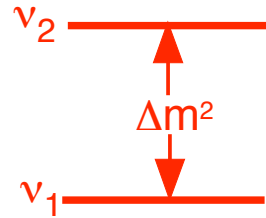
$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \cong S_{\alpha\beta} \sin^2\left(\Delta m^2 \frac{L}{4E}\right); \quad S_{\alpha\beta} \equiv 4 \left| \sum_{i \text{ Clump}} U_{\alpha i}^* U_{\beta i} \right|^2.$$

For no flavor change,

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \cong 1 - 4T_\alpha(1 - T_\alpha) \sin^2\left(\Delta m^2 \frac{L}{4E}\right); \quad T_\alpha \equiv \sum_{i \text{ Clump}} |U_{\alpha i}^*|^2.$$

“i Clump” is a sum over only the mass eigenstates on one end of the big gap Δm^2 .

When There are Only Two Flavors and Two Mass Eigenstates



$$U = \begin{matrix} \nu_\alpha \\ \nu_\beta \end{matrix} \begin{bmatrix} \nu_1 & \nu_2 \\ \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} ; \quad S_{\alpha\beta} = 4T_\alpha(1 - T_\alpha) = \sin^2 2\theta$$

Mixing angle

For $\beta \neq \alpha$,
$$P(\nu_\alpha^{(-)} \leftrightarrow \nu_\beta^{(-)}) = \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right) .$$

For no flavor change,
$$P(\nu_\alpha^{(-)} \rightarrow \nu_\alpha^{(-)}) = 1 - \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right) .$$