Double beta decay: Theory

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$\mathcal{I}$.

Introduction

$\beta\beta$ decay modes

$$(Z, A) \Rightarrow (Z \pm 2, A) + 2e^{\mp} + X$$

$\beta\beta$ decay modes

$$(Z, A) \Rightarrow (Z \pm 2, A) + 2e^{\mp} + 2\nu$$

$2\nu\beta\beta$ decay:

$\rightarrow$ allowed within Standard model,
2nd order process in Fermi theory

$\rightarrow$ observed for 11 isotopes:
^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo,
^{116}Cd, ^{128,130}Te, ^{150}Nd and ^{238}U
First double beta plus decay: ^{130}Ba

$\rightarrow$ $T_{1/2}^{2\nu\beta\beta} \sim 10^{(19-25)}$ ys $\qquad (\sim G_F^4)$

$\rightarrow$ Important constraint for nuclear matrix element calculation

$\beta\beta$ decay modes

$$(Z, A) \Rightarrow (Z \pm 2, A) + 2e^{\mp}$$

$0\nu\beta\beta$ decay:

$\rightarrow$ violates lepton number by 2 units

$\rightarrow$ experimentally not observed

$\rightarrow$ $T_{1/2}^{0\nu\beta\beta}(^{76}Ge) \gtrsim 10^{25}$ ys

$\rightarrow$ Current bounds limit neutrino mass scale to $m_\nu \leq \mathcal{O}(0.5 - 1)$ eV

$\rightarrow$ Observation implies physics beyond the standard model of particle physics

$\beta\beta$ decay modes

$$(Z,A) \Rightarrow (Z \pm 2, A) + 2e^{\mp} + X$$

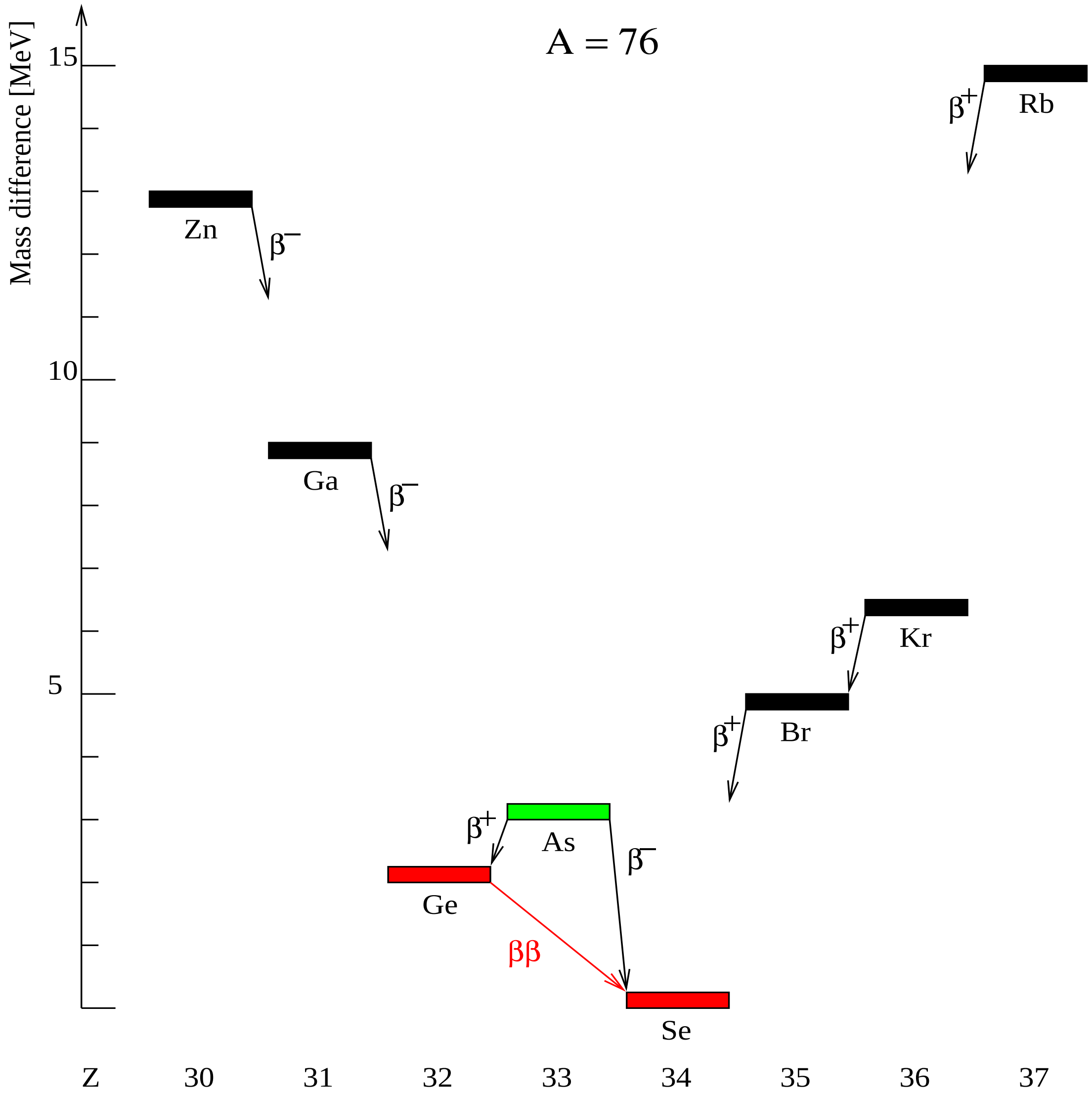
"Exotic" decays:

$\rightarrow$ for example X = J, i.e. Majoron

$\rightarrow$ experimentally not observed (and no rumours!)

$\rightarrow$ Best limit from: $T_{1/2}^{0\nu\beta\beta}(^{128}Te) \gtrsim$ few 10^{24} ys

$\beta\beta$-decay "candidates"



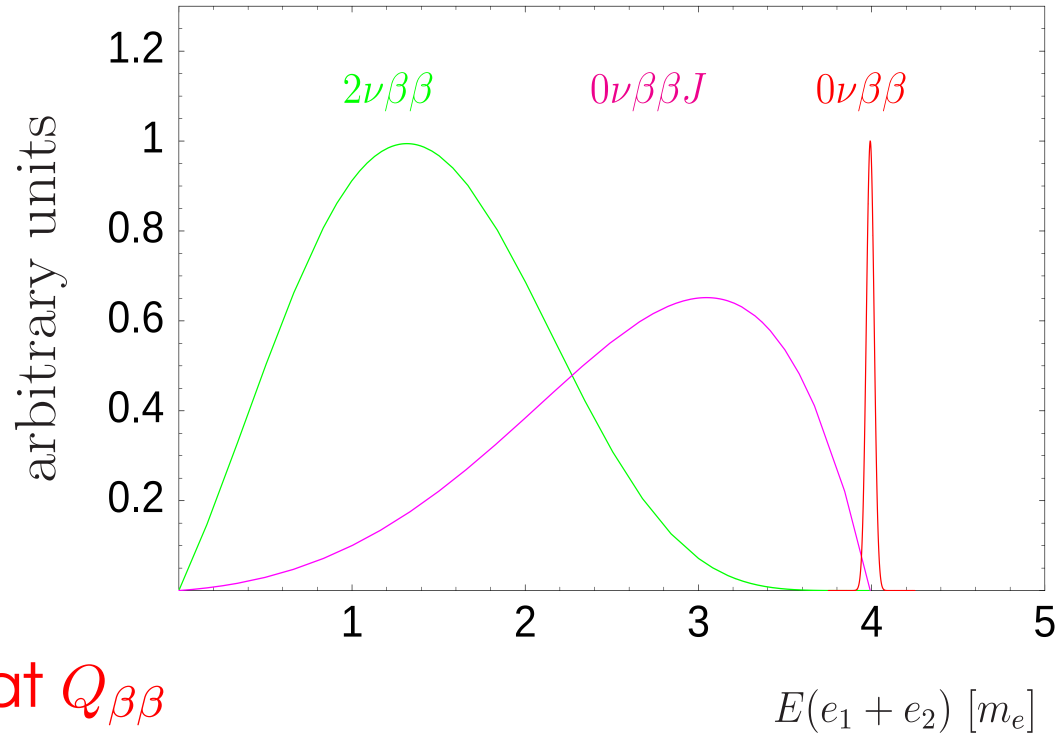
^{76}Ge stable against β^- and β^+/EC decay

Even-even isotopes have lower ground states: pairing force!

35 $\beta^-\beta^-$ and 30 EC/EC candidates on the chart of nuclei

Distinguish decay modes?

⇒ Neutrinos escape unmeasured, $2\nu\beta\beta$ decay continuum

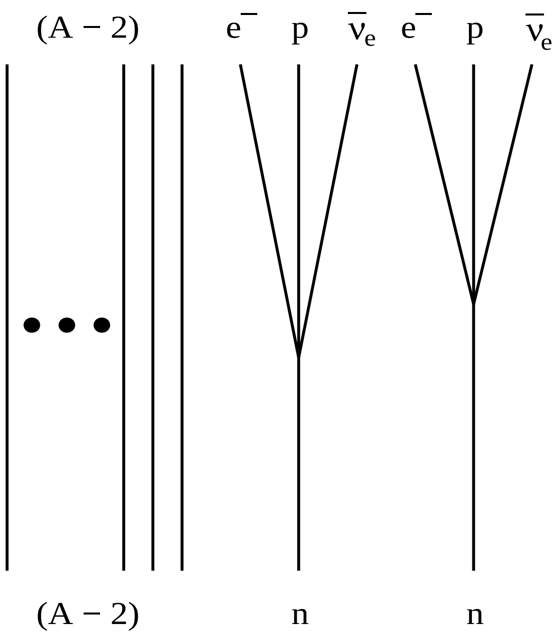


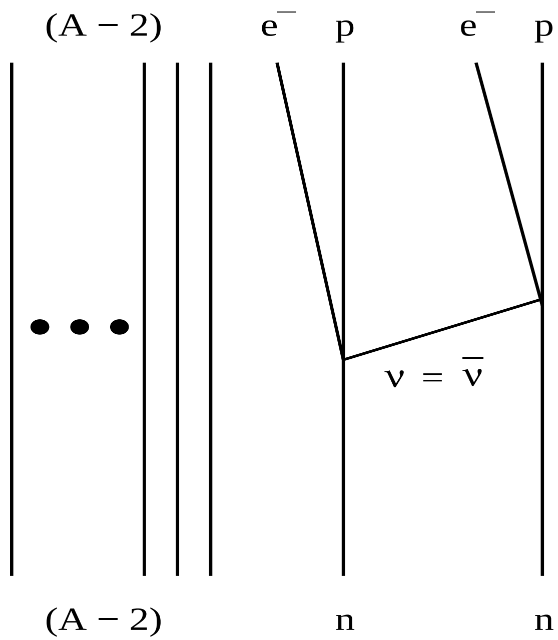
⇒ Only electrons carry energy: $0\nu\beta\beta$ decay peak at $Q_{\beta\beta}$

⇒ Good energy resolution essential for experiments: $2\nu\beta\beta$ decay is irreducible background for $0\nu\beta\beta$ decay experiments

$2\nu\beta\beta$ & $0\nu\beta\beta$ *schematically*

$2\nu\beta\beta$ decay	$0\nu\beta\beta$ decay





Standard model allowed
2nd order process:
Amplitude $\sim G_F^2$

Only possible if:
$\nu = \nu^c$
Amplitude $\sim m_\nu G_F^2$

Half-life

$2\nu\beta\beta$ decay:

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = G_{2\nu}|M_{GT}^{2\nu\beta\beta}|^2$$

$G_{2\nu}$ - phase space integral
$M_{GT}^{2\nu\beta\beta}$ - nuclear structure matrix element

$0\nu\beta\beta$ decay (mass mechanism):

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{0\nu}|\langle m_\nu\rangle|^2|M_{m_\nu}^{0\nu\beta\beta}|^2$$

$G_{0\nu}$ - phase space integral
$M_{m_\nu}^{0\nu\beta\beta}$ - nuclear structure matrix element. Note: $M_{m_\nu}^{0\nu\beta\beta} \neq M_{GT}^{2\nu\beta\beta}$
$\langle m_\nu\rangle$ - effective Majorana neutrino mass

$\Rightarrow$ Phase space can be calculated easily

$\Rightarrow$ Uncertainty completely dominated by $M_{GT}^{2\nu\beta\beta}$, $M_{m_\nu}^{0\nu\beta\beta}$

Phase space integrals

Example $2\nu\beta\beta$ decay: Haxton & Stephenson, PPANP 12 (1984) 409

$$G_{2\nu} = \frac{a_{2\nu}}{\ln(2)} \int_{m_e}^{W_0 - m_e} k_1 \epsilon_1 F(Z, \epsilon_1) d\epsilon_1 \int_{m_e}^{W_0 - \epsilon_1} k_2 \epsilon_2 F(Z, \epsilon_2) d\epsilon_2 \int_0^{W_0 - \epsilon_1 - \epsilon_2} \nu_1^2 \nu_2^2 d\nu_1$$

$a_{2\nu}$ - physical constants

ϵ_i, k_i and ν_i - energy and momenta of electrons and energy of neutrinos

$F(Z, \epsilon_i)$ - Coulomb corrections

Primakoff-Rosen approximation:

$$F(Z, \epsilon_i)_{\mp} = \pm \frac{\epsilon_i}{k_i} \frac{2\pi\alpha Z}{1 - e^{\mp 2\pi\alpha Z}}$$

$\Rightarrow$ allows to solve integrals analytically, simple polynomial in $\tilde{T}_0 = (T_0/m_e) = (W_0 - 2m_e)/m_e$

$\Rightarrow$ quick estimate, explains shape of spectrum

$\Rightarrow$ "exact" numerical integration, see Doi, Kotani & Takasugi, Prog. Th. Phys. Suppl. 83 (1985) 1

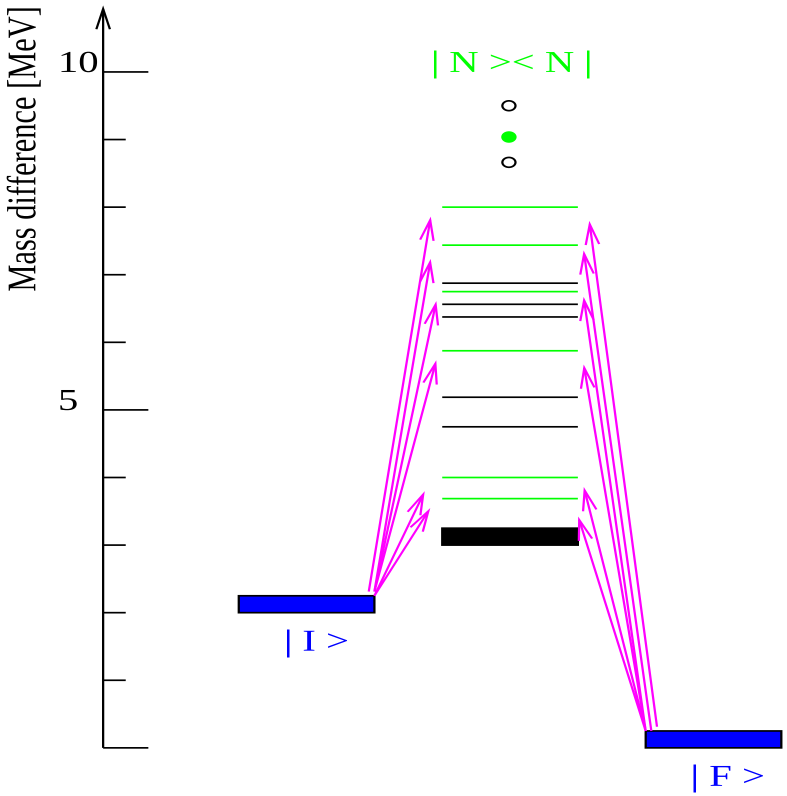
$\mathcal{II}.$

Nuclear matrix elements

$2\nu\beta\beta$ decay matrix element

$$\mathcal{M}^{GT}_{2\nu\beta\beta} = \sum_N \frac{\langle F||\tau_-\sigma||N\rangle\langle N||\tau_-\sigma||I\rangle}{E_N + T_0/2 + m_e - E_I}$$

Graphically:



$2\nu\beta\beta$ versus $0\nu\beta\beta$

$2\nu\beta\beta$: $\mathcal{M}^{GT,c}_{2\nu\beta\beta} = \langle F||(\tau_-\sigma) < E > (\tau_-\sigma)||I\rangle$ Only 1^+ Only GT

$0\nu\beta\beta$: $\mathcal{M}^{GT,c}_{0\nu\beta\beta} = \langle F||(\tau_-\sigma) H_m(r_{ij})(\tau_-\sigma)||I\rangle$ All J^π + Also $F, T, \cdots$

with:

$$< E > = \frac{1}{\langle E_N\rangle + T_0/2 + m_e - E_I}$$

$$H_m(r_{ij}) = \frac{R}{2\pi^2}\int \frac{d\mathbf{q}}{\omega} \frac{e^{i\mathbf{q}\cdot r_{ij}}}{(\omega + T_0/2 + m_e + \langle E_N\rangle - E_I)}$$

Here, the so-called "closure approximation" has been used:

$$\sum_N \frac{||N\rangle\langle N|}{E_N + T_0/2 + m_e - E_I} = \frac{1}{\langle E_N\rangle + T_0/2 + m_e - E_I}$$

Note that this is a reliable aprroximation for $0\nu\beta\beta$ decay, but not so good in $2\nu\beta\beta$ decay.

Nuclear models

Basically nowadays two microscopic approaches to the nuclear many body problem are used in double beta decay:

ISM: Interacting Shell Model

For a recent review see: E. Caurier et al, Rev. Mod. Phys. 77:427-488,2005; (nucl-th/0402046)

pn-QRPA: proton-neutron Quasiparticle Random Phase Approximation

See, for example: P. Ring and P. Schuck, "The nuclear many body problem" (Springer, New York, 1980); explicit references for pn-QRPA in $\beta\beta$, see below

Interacting Shell Model

All microscopic calculations based on the Independent Particle shell model (IPM) Starting point: Nucleons moving independently in a mean field:

$$U(r) = \frac{1}{2}\hbar\omega r^2 + D\vec{l}^2 + C\vec{l}\vec{s}$$

With (strongly attractive) spin-orbit term, able to explain magic numbers. Interactions between nucleons described by residual interactions:

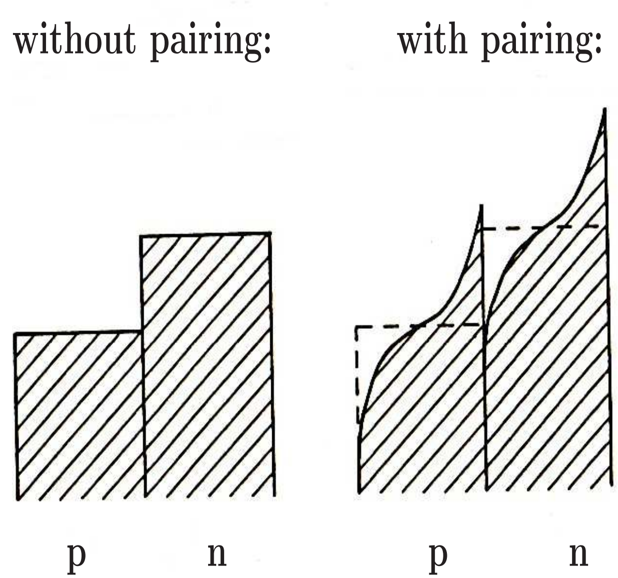
$$\mathcal{H} = \sum_{ij} \mathcal{K}_{ij} a_i^\dagger a_j - \sum_{i\le j, k\le l} \mathcal{V}_{ijkl} a_i^\dagger a_j^\dagger a_k a_l$$

$\Rightarrow$ Conceptually "simple": Given a "good enough" residual interaction, $\mathcal{V}_{ijkl}$ the problem is reduced to diagonalizing a matrix in a sufficiently large model space

pn-QRPA: Pairing

Basic idea:

Most important part of residual interaction is the pairing force



Occupation probability
instead of:
$v^2 = 1$ (filled)
$v^2 = 0$ (empty)

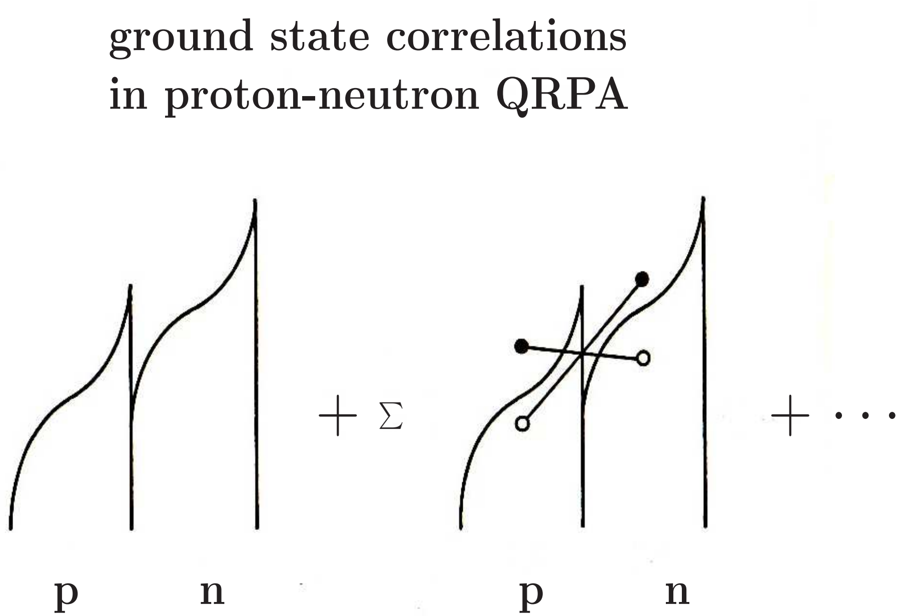
⇒ Use BCS theory for pairing: Ground state consists of pairs of like-wise nucleons with $J^\pi = 0^+$

⇒ Change to quasi-particle basis:
Ground state is the vacuum of quasi-particles

⇒ Quasi-particle = a · Particle + b · Hole with $a^2+b^2=1$

pn-QRPA

Random Phase Approximation:

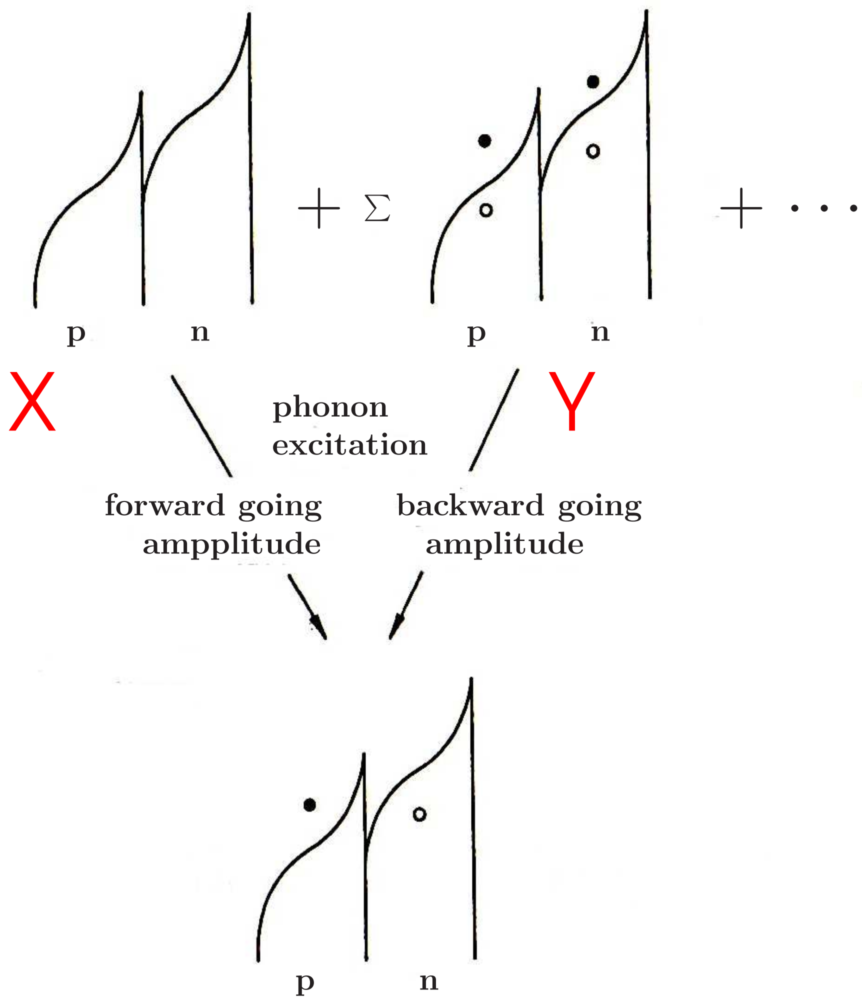


$\Rightarrow |RPA\rangle = |BCS\rangle \quad + \quad \sum |2p2h\rangle + \cdots$

$\Rightarrow$ 2-Particle-2-Hole excitations lowest order correction

$\Rightarrow$ If ground state correlations look “randomly” distributed,
RPA is good approximation

QRPA Phonons



Charge-changing transition
in pn-QRPA:
particle-hole pair = phonon

Creation of ph-pair
from $|BCS\rangle$
or destruction of ph-pair
from $|2p2h\rangle$
can lead to same
final state

Assumption:
$Y^2 \ll X^2$

$\Rightarrow$ Creation of phonon from $|BCS\rangle$ only overstimates transition probability

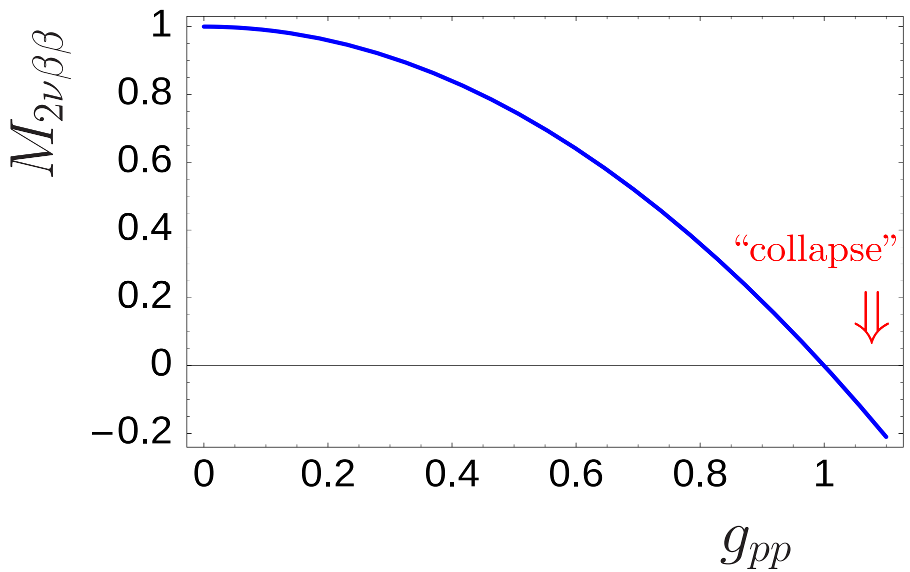
$\Rightarrow$ particle-particle interactions enhance Y, reduce transition probability

pn-QRPA versus ISM

Comment:	pn-QRPA	-/+	ISM	-/+
Correlations	“lowest order”	-	everything	+
Model space	large	+	small	–
Parameter dependence	g_{pp} (strong)	-	???	
Beyond closure	yes	+	no	-
Future improvements?	maybe		yes	++

pn-QRPA: g_{pp} Problem

Dependence of $2\nu\beta\beta$ decay matrix element on g_{pp} schematically:



This figure not a real calculation!

$g_{pp} \simeq 1$ realistic + suppression of $M_{2\nu\beta\beta}$ achieved, but:

$\Rightarrow$ $Y \sim X$ at $g_{pp} \geq 1$, pn-QRPA "collapses"

$\Rightarrow$ Strong dependence of $M_{2\nu\beta\beta}$ near $g_{pp} \simeq 1$, predictions difficult

$\Rightarrow$ "Fortunately" $M_{0\nu\beta\beta}$ less dependent on g_{pp}

$T_{1/2}^{2\nu\beta\beta}$: Test matrix elements

	Theory	Exp	NEMO-3	$\Delta\mathcal{M}$	Comment
^{76}Ge (10^{21} y)	3.0	(1.3 $\pm$ 0.1)		1.5	
^{82}Se (10^{20} y)	1.1	(0.92$\pm$ 0.1)	(0.96 $\pm$ 0.1)	1.1	no pred.
^{96}Zr (10^{19} y)	1.1	(1.4$^{+3.5}_{-0.5}$)	(2.0 $\pm$ 0.4)	0.89	
^{100}Mo (10^{18} y)	1.1	(8.0 $\pm$ 0.6)	(7.1 $\pm$ 0.55)	0.37	
^{116}Cd (10^{19} y)	6.3	(3.2 $\pm$ 0.3)	(2.8 $\pm$ 0.3)	1.41	
^{128}Te (10^{24} y)	2.6	(7.2 $\pm$ 0.3)		0.60	no pred.
^{130}Te (10^{21} y)	1.8	(2.7 $\pm$ 0.1)	(0.76 $\pm$ 0.07)	0.82	no pred.
^{150}Nd (10^{18} y)	7.4	(7.0$^{+11}_{-0.3}$)	(9.7 $\pm$ 1.2)	1.03	

Theory: pn-QRPA K. Muto et al., Z.Phys. A 334 (1989) 177

Experimental data from review: S. R. Elliott and P. Vogel,
Ann. Rev. Nucl. Part. Sci. **52**, 115 (2002)

NEMO-3: L. Vála for the NEMO Collaboration, arXiv:0710.5604

Definition of $\Delta\mathcal{M} = \mathcal{M}^{\rm exp}/\mathcal{M}^{\rm cal}$ relativ to Exp

Average deviation for 8 (5) isotopes only 50 (60) % !

Double electron capture

A.P. Meshik et al., PRC64 (2001) 035205 (geochemical):

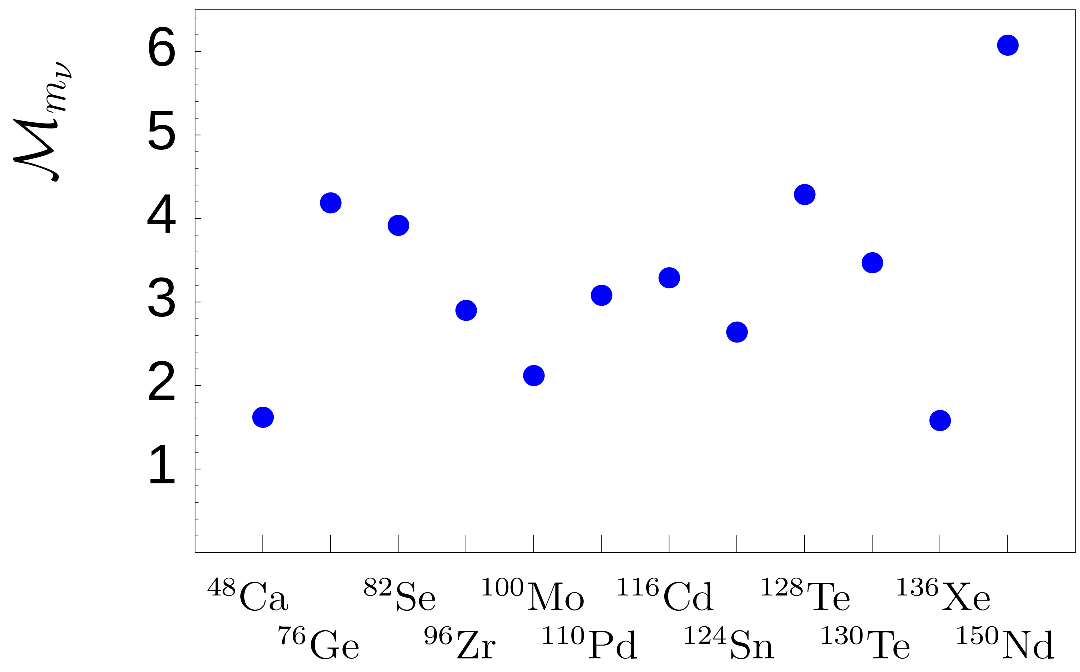
$T_{1/2}^{\beta\beta}(^{130}Ba) = (2.2 \pm 0.5) \times 10^{21}$ y

M. Hirsch et al., Z. Phys. A347 (1994) 151 theoretical prediction:

$T_{1/2}^{2\nu EC/EC}(^{130}Ba) = 4.2 \times 10^{21}$ y

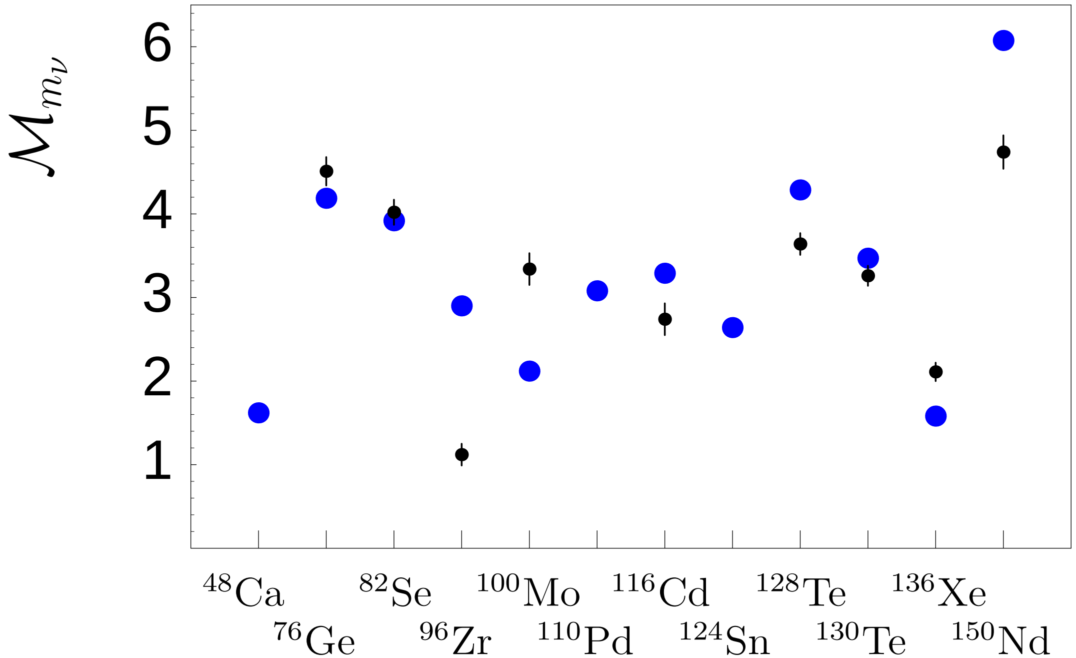
$\Rightarrow \Delta\mathcal{M} = 1.38$

$0\nu\beta\beta$ M.E. uncertainty



blue (pn-QRPA): K. Muto et al., Z.Phys. A 334 (1989) 187; Z.Phys. A339 (1991) 435; Europhys. Lett. 13 (1990) 31

$0\nu\beta\beta$ M.E. uncertainty

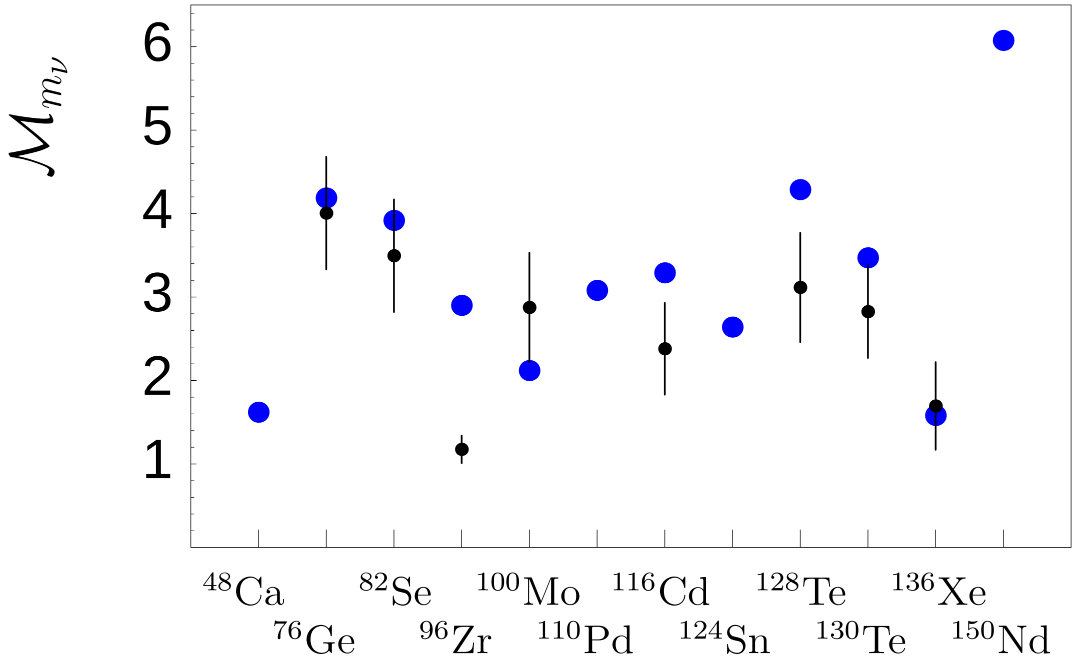


Error bars:
only Δg_{pp}
from $2\nu\beta\beta$ fit

blue (pn-QRPA): K. Muto et al., Z.Phys. A 334 (1989) 187; Z.Phys. A339 (1991) 435; Europhys. Lett. 13 (1990) 31

black (pn-QRPA): V. Rodin et al., Nucl.Phys. A766 (2006) 107 Erratum A793:213-215,2007

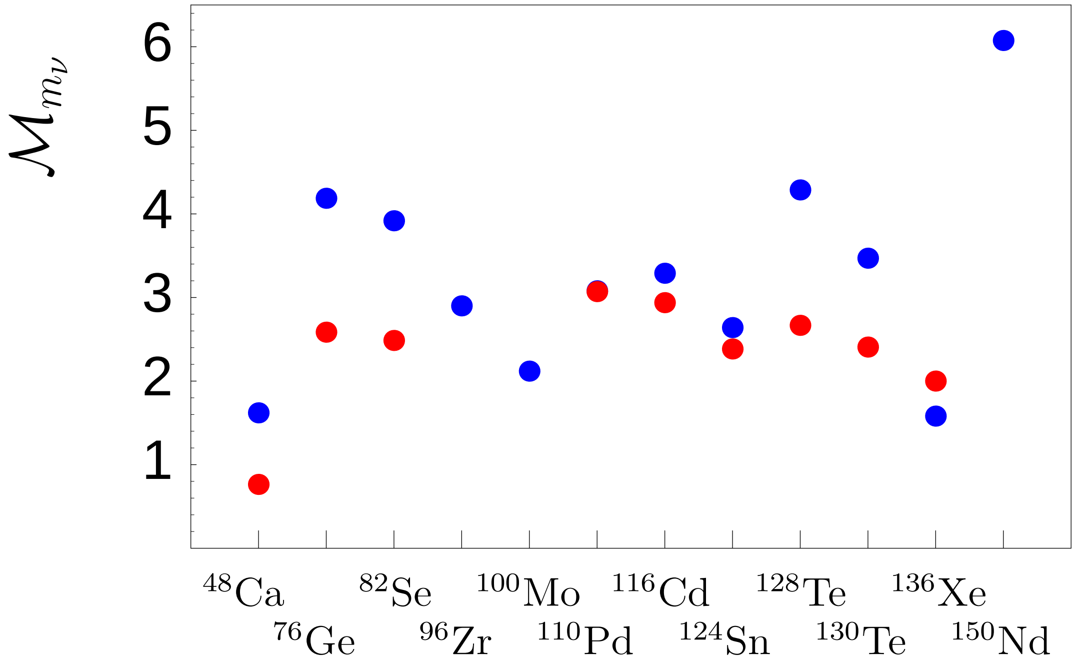
$0\nu\beta\beta$ M.E. uncertainty



Error bars:
from averaging
20 calculations
QRPA + RQRPA
$V_{\rm res}$'s
+ model spaces
+Δg_{pp}+ SRC

blue (pn-QRPA): K. Muto et al., Z.Phys. A 334 (1989) 187; Z.Phys. A339 (1991) 435; Europhys. Lett. 13 (1990) 31
black (pn-QRPA): F. Simkovic et al., arXiv:0710.2055v3 (2 April 2008)

$0\nu\beta\beta$ M.E. uncertainty



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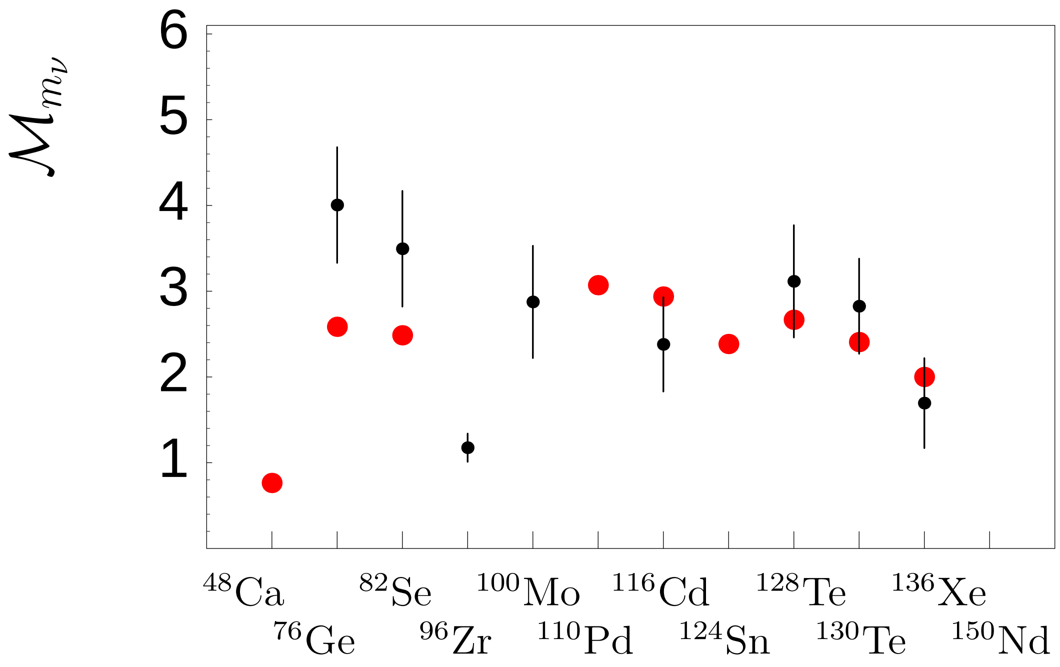
red (ISM): E. Caurier et al.; Eur. Phys. J. A36 (2008) 195 (arXiv:0709.0277)

$0\nu\beta\beta$ M.E. uncertainty



Error bars:
from averaging
20 calculations
QRPA + RQRPA
+ model spaces
$+\Delta g_{pp}$+ SRC

red (ISM): E. Caurier et al.; Eur. Phys. J. A36 (2008) 195 (arXiv:0709.0277)
black (pn-QRPA): F. Simkovic et al., arXiv:0710.2055v3 (2 April 2008)

$0\nu\beta\beta$ M.E.: Summary

Good news:

$\Rightarrow$ Latest calculations agree
within a factor of 2 (or better)

$\Rightarrow$ Improvements possible in the future (ISM)

Bad news:

$\Rightarrow$ ISM is (systematically?) lower than pn-QRPA

$\Rightarrow$ Disagreement in many details (SRC, HOC, χ_F, etc...)

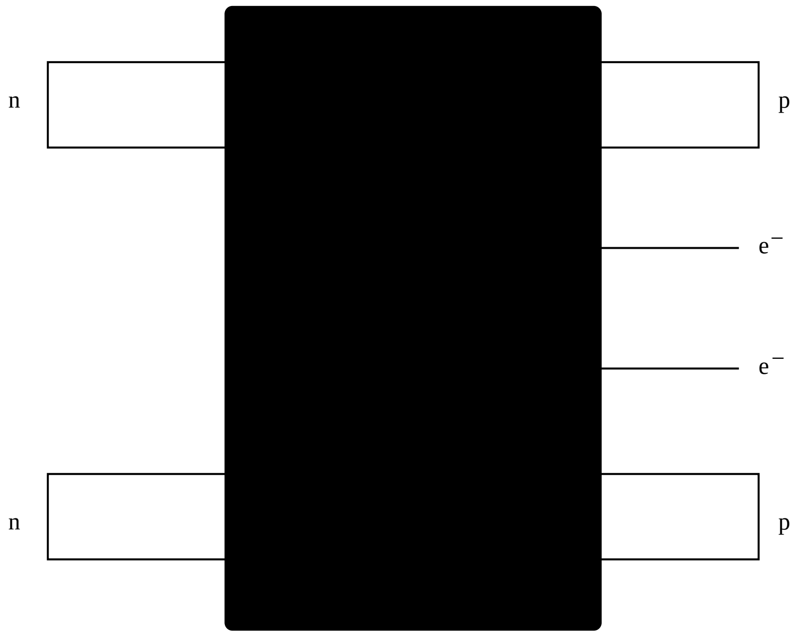
$\Rightarrow$ Only $2\frac{1}{2}$ groups working on the problem

$\mathcal{III}.$

Mass mechanism

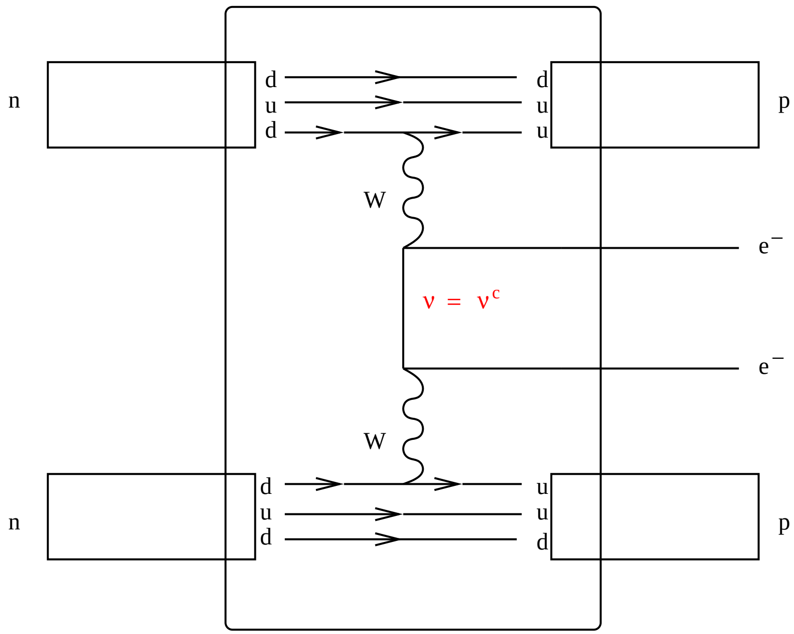
Mass mechanism

Simplest possibility for a $0\nu\beta\beta$ diagram:



Mass mechanism

Simplest possibility for a $0\nu\beta\beta$ diagram:



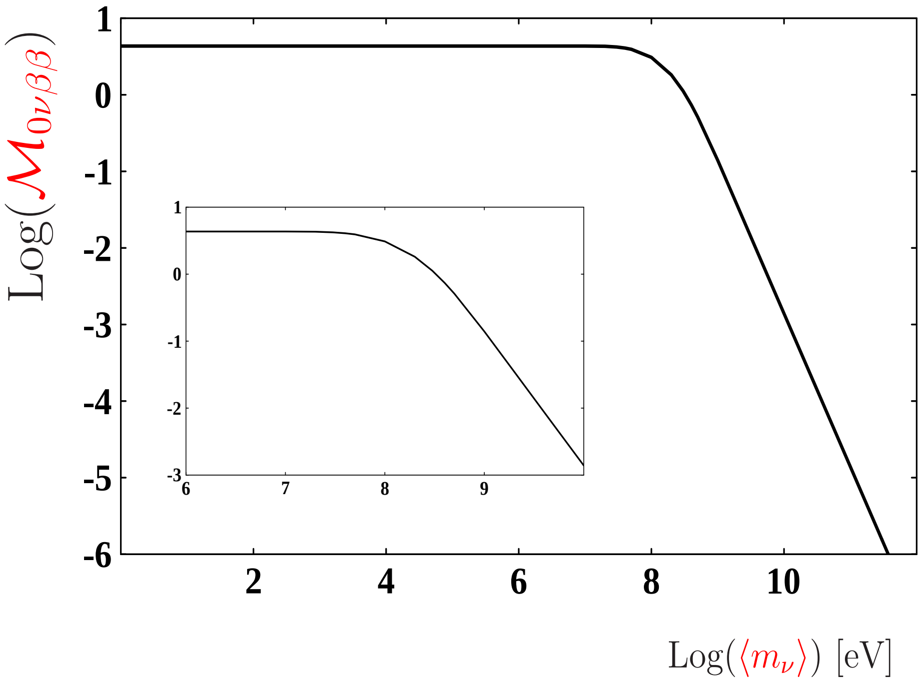
Neutrino propagator:

$$\int \frac{d^4p}{(2\pi)^4}\frac{m_\nu + \not{p}}{p^2 - m_\nu^2}$$

"Mass mechanism" because weak interaction is left-handed:

$$P_L(m_\nu + \not{p}\ )P_L = m_\nu P_L$$

$\mathcal{M}_{0\nu\beta\beta}$ as function of $\langle m_\nu \rangle$

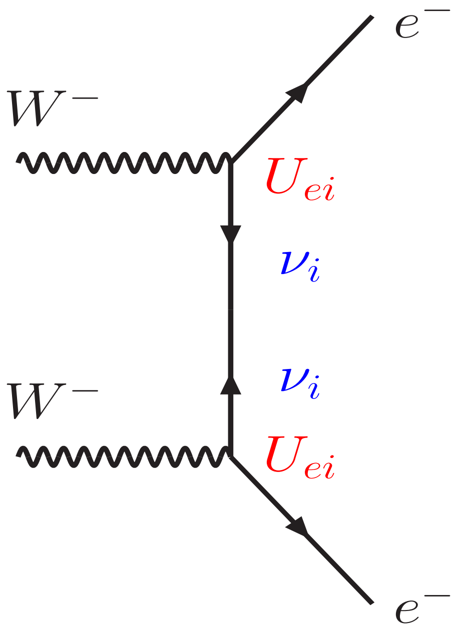


Separate nuclear and particle physics:

Constant for small m_ν $\Rightarrow$ $T_{1/2} \sim m_\nu^{-2}$

$(\sim 1/m_\nu)^2$ for large m_ν $\Rightarrow$ $T_{1/2} \sim m_\nu^2$

Neutrino mixing



Each vertex:

$$W_\mu^- \bar{e}\gamma^\mu P_L U_{ei}\nu_i$$

Full propagator reads:

$$U_{ei}^2 \int \frac{d^4p}{(2\pi)^4} \frac{m_{\nu_i} + \not{p}}{p^2 - m_{\nu_i}^2}$$

Define in the limit of small neutrino masses:

$$\langle m_\nu \rangle = \sum_i U_{ei}^2 m_{\nu_i}$$

$\langle m_\nu \rangle$ and ν spectrum

Neutrinos mix, thus:

$$\begin{aligned} \langle m_\nu \rangle &= \sum_j U_{ej}^2 m_j \\ &= c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3 \end{aligned}$$

A priori seven unknown quantities:

$\Rightarrow$ 3 masses: m_i

$\Rightarrow$ 2 angles: θ_{12} and θ_{13}

$\Rightarrow$ 2 CP violating phases: α and β

$\langle m_\nu \rangle$ and ν spectrum

Neutrinos mix, thus:

$$\begin{aligned}\langle m_\nu \rangle &= \sum_j U_{ej}^2 m_j \\ &= c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3\end{aligned}$$

+ Neutrino oscillation data:

$\Rightarrow$ 1 mass: m_{ν_1} + $\Delta m^2_{\rm Atm}$, $\Delta m^2_\odot$

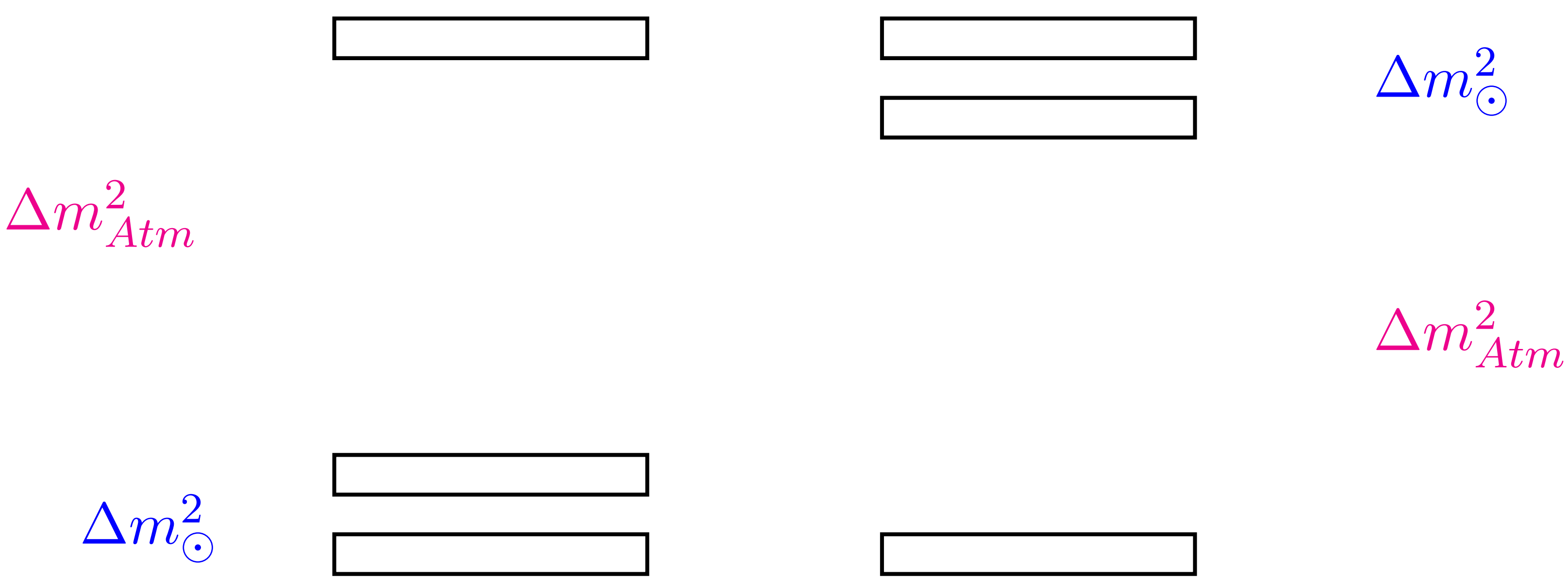
$\Rightarrow$ 2 angles: $\theta_\odot$ and θ_R

$\Rightarrow$ 2 CP violating phases: α and β

Inverse or normal hierarchy?

Both of the following can solve solar and atmospheric neutrino problems:

Normal hierarchy | Inverse hierarchy



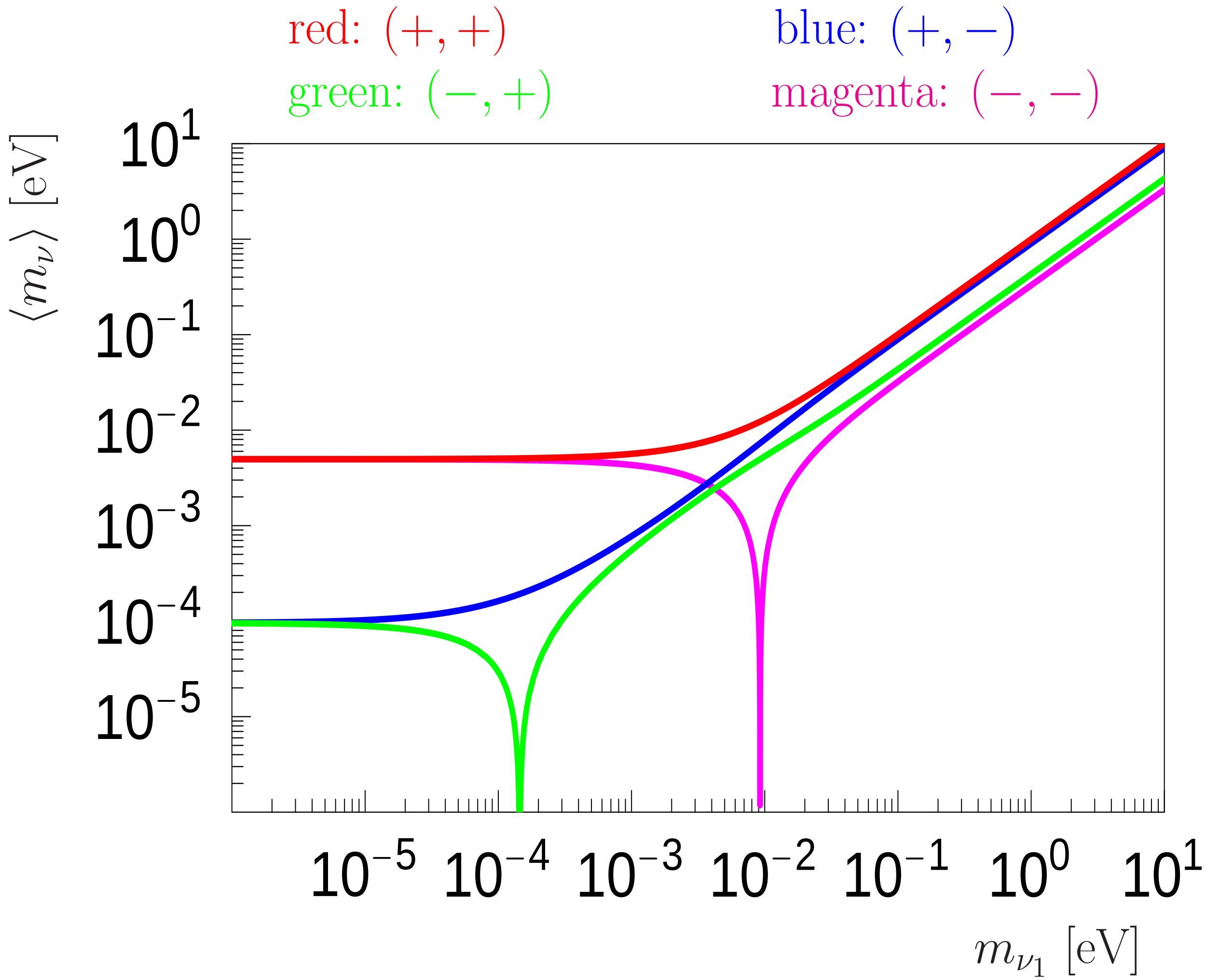
Expectations for $\langle m_\nu \rangle$?

Current oscillation experiments can not tell difference between normal and inverse hierarchy. What is the expectation for $0\nu\beta\beta$?

Normal hierarchy: $\langle m_\nu \rangle \simeq s_{12}^2 \sqrt{\Delta m_\odot^2} \simeq 3 \times 10^{-3}$ eV

Inverse hierarchy: $\langle m_\nu \rangle \simeq \sqrt{\Delta m_{Atm}^2} \simeq 5 \times 10^{-2}$ eV

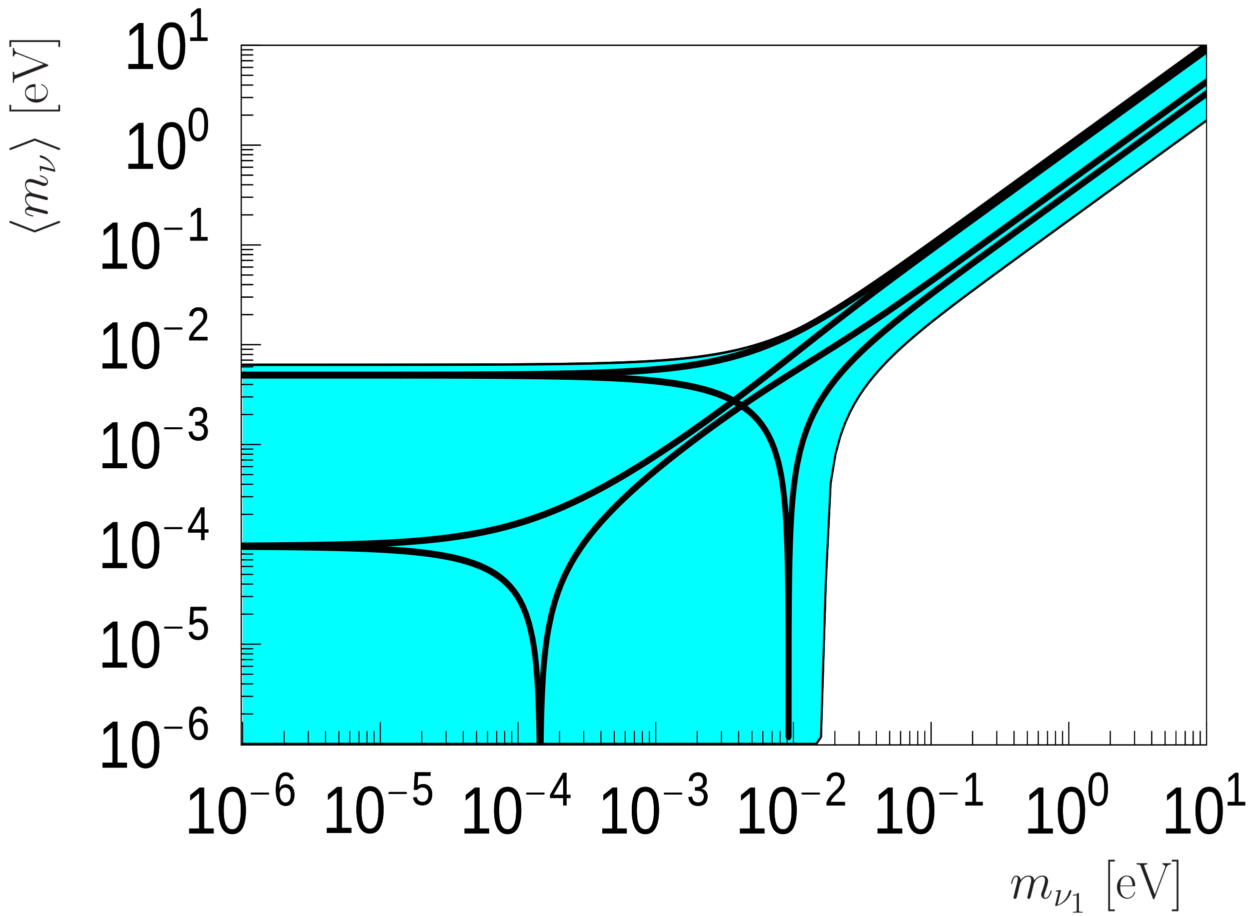
$0\nu\beta\beta$ and ν oscillations



$$\Delta m^2_{Atm} = 2.2 \cdot 10^{-3}\ \text{eV}^2, \qquad \Delta m^2_{\odot} = 8.1 \cdot 10^{-5}\ \text{eV}^2,$$

$$\sin^2\theta_{\odot} = 0.3, \qquad \sin^2\theta_R = 0.051$$

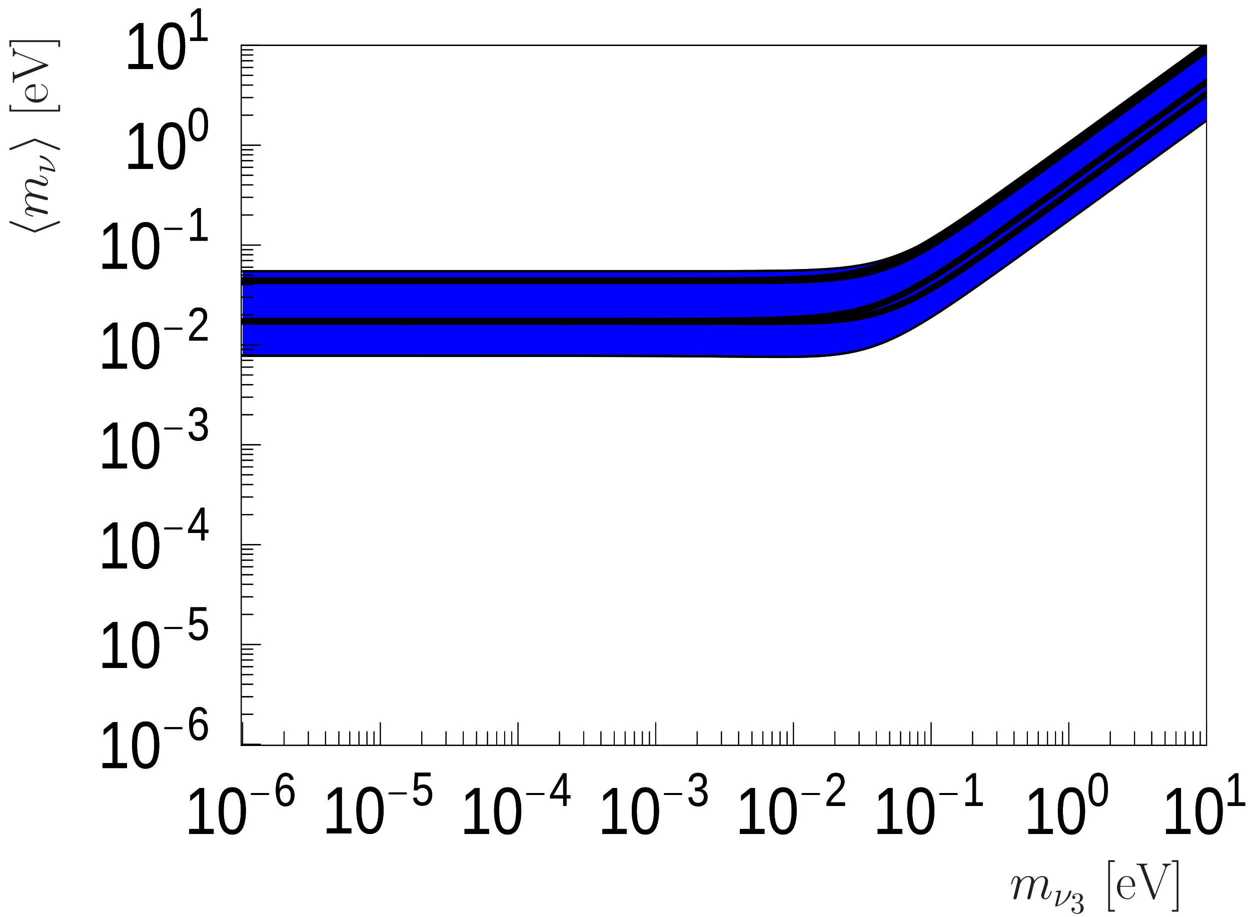
$0\nu\beta\beta$ and ν oscillations



$\Delta m^2_{Atm} = [1.4, 3.3] \cdot 10^{-3}$ eV^2, $\quad \Delta m^2_{\odot} = [7.2, 9.1] \cdot 10^{-5}$ eV^2,

$\sin^2\theta_{\odot} = [0.23, 0.38]$, $\quad \sin^2\theta_R = [0, 0.051]$

Inverse hierarchy



$$\Delta m^2_{Atm} = [1.4, 3.3] \cdot 10^{-3} \text{ eV}^2, \qquad \Delta m^2_{\odot} = [7.2, 9.1] \cdot 10^{-5} \text{ eV}^2,$$

$$\sin^2\theta_{\odot} = [0.23, 0.38], \qquad \sin^2\theta_R = [0, 0.051]$$

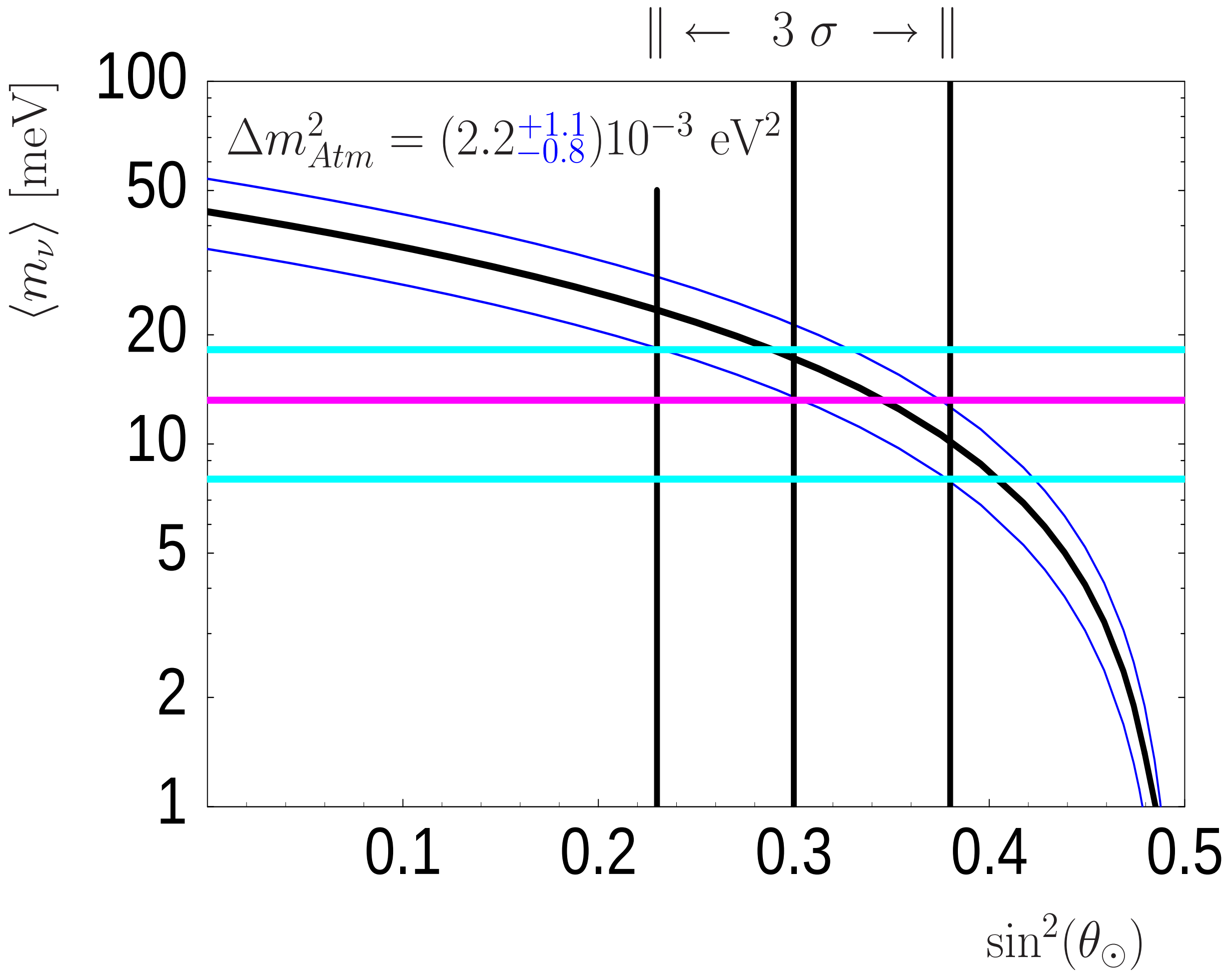
Lower limit - inverse hierarchy

Recall:

$$\begin{aligned} \langle m_\nu \rangle &= \sum_j U_{ej}^2 m_j \\ &\simeq c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 \\ &\sim (c_\odot^2 - s_\odot^2)\sqrt{\Delta m^2_{Atm}} \\ &\simeq 0.4 \cdot \sqrt{2.2 \cdot 10^{-3}}\ \text{eV} \quad \simeq 19\ \text{meV} \end{aligned}$$

$\Rightarrow$ Lower limit exists, if $\theta_\odot$ non-maximal

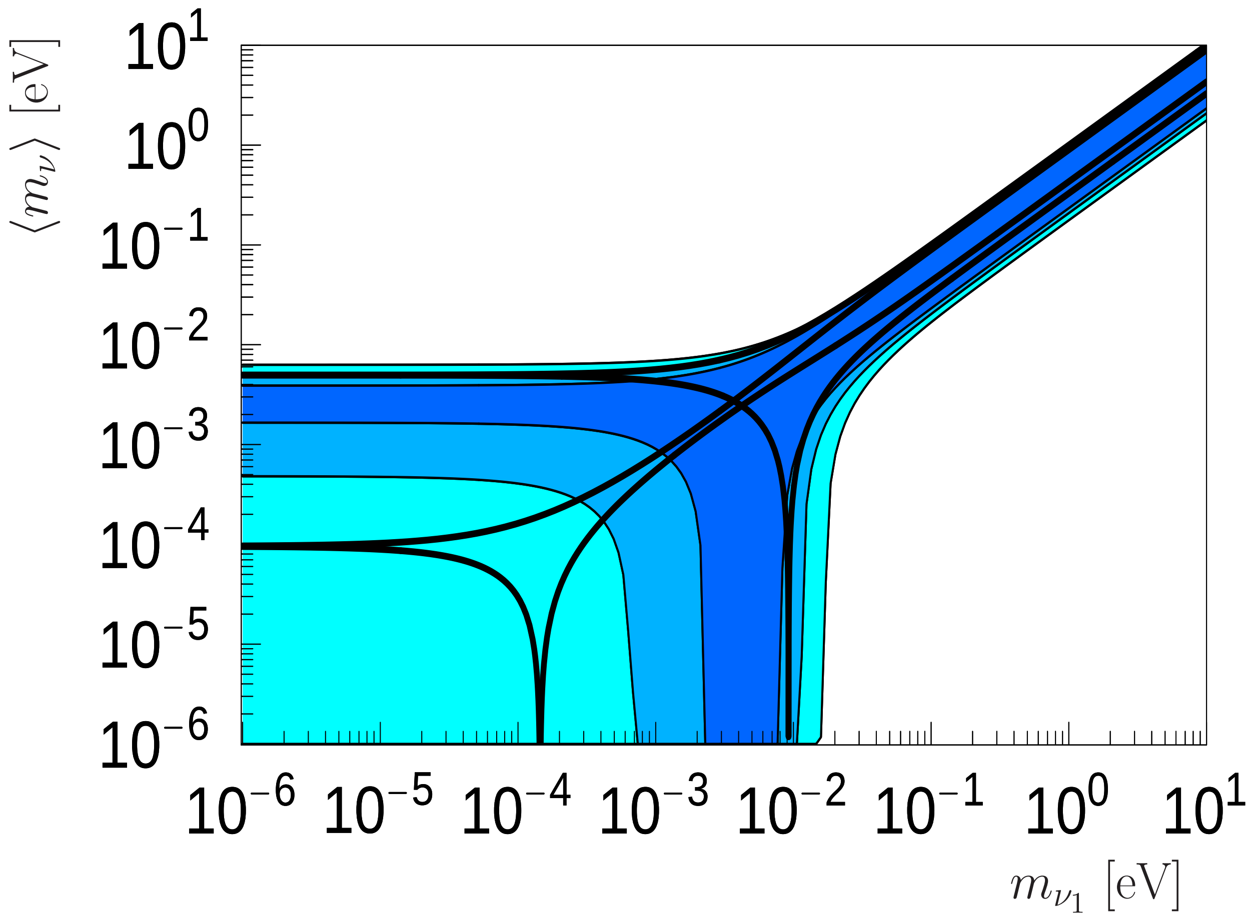
Future lower limit?



Current lower limit @ 3 σ c.l.: $\langle m_\nu \rangle \simeq 7.5$ meV

@ 2 σ c.l.: $\langle m_\nu \rangle \simeq 12$ meV

Reactor angle and $0\nu\beta\beta$



$\sin^2\theta_R =$

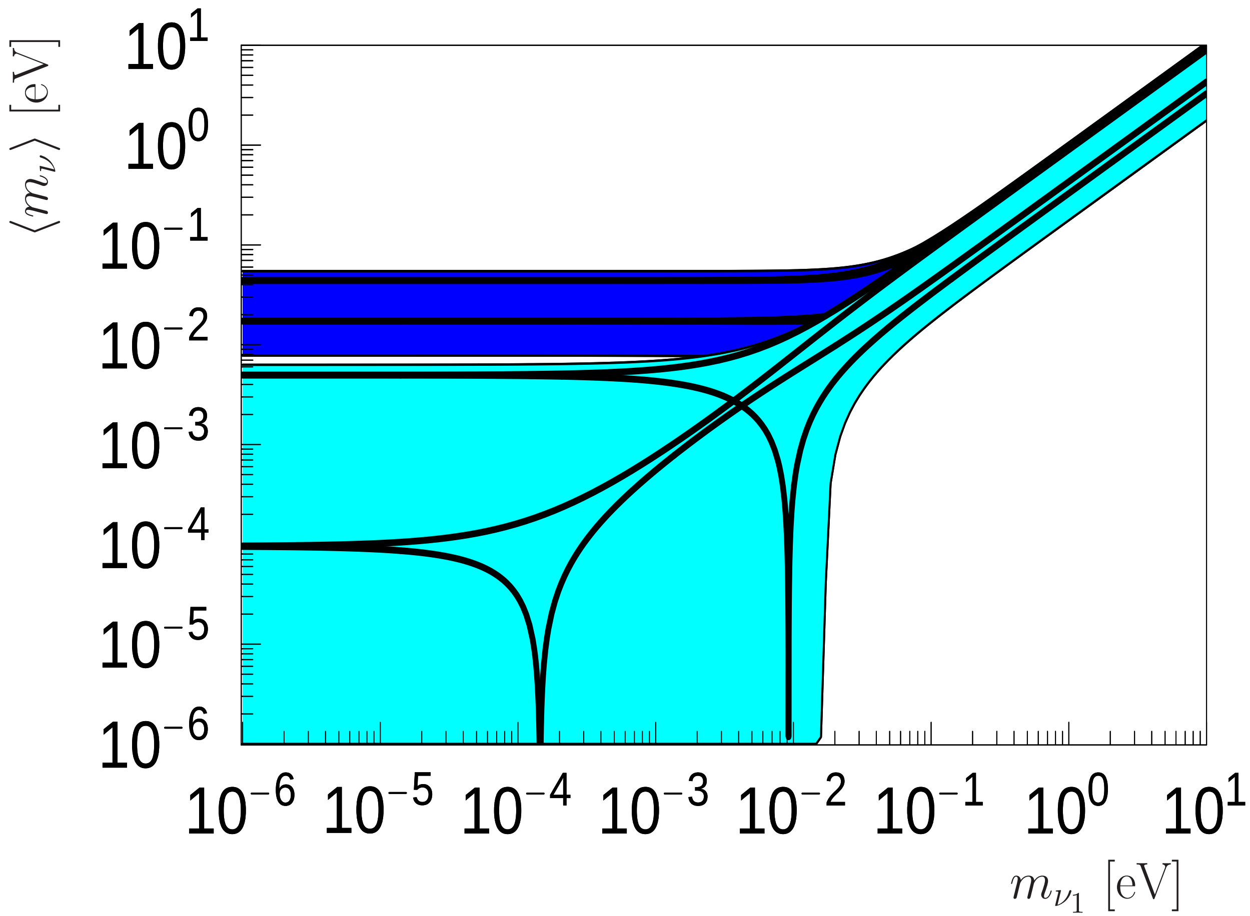
$[0, 0.051]$

$[0, 0.025]$

$[0, 0.0051]$

$\Delta m^2_{Atm} = [1.4, 3.3] \cdot 10^{-3}$ eV^2, $\Delta m^2_{\odot} = [7.2, 9.1] \cdot 10^{-5}$ eV^2,
$\sin^2\theta_{\odot} = [0.23, 0.38]$,

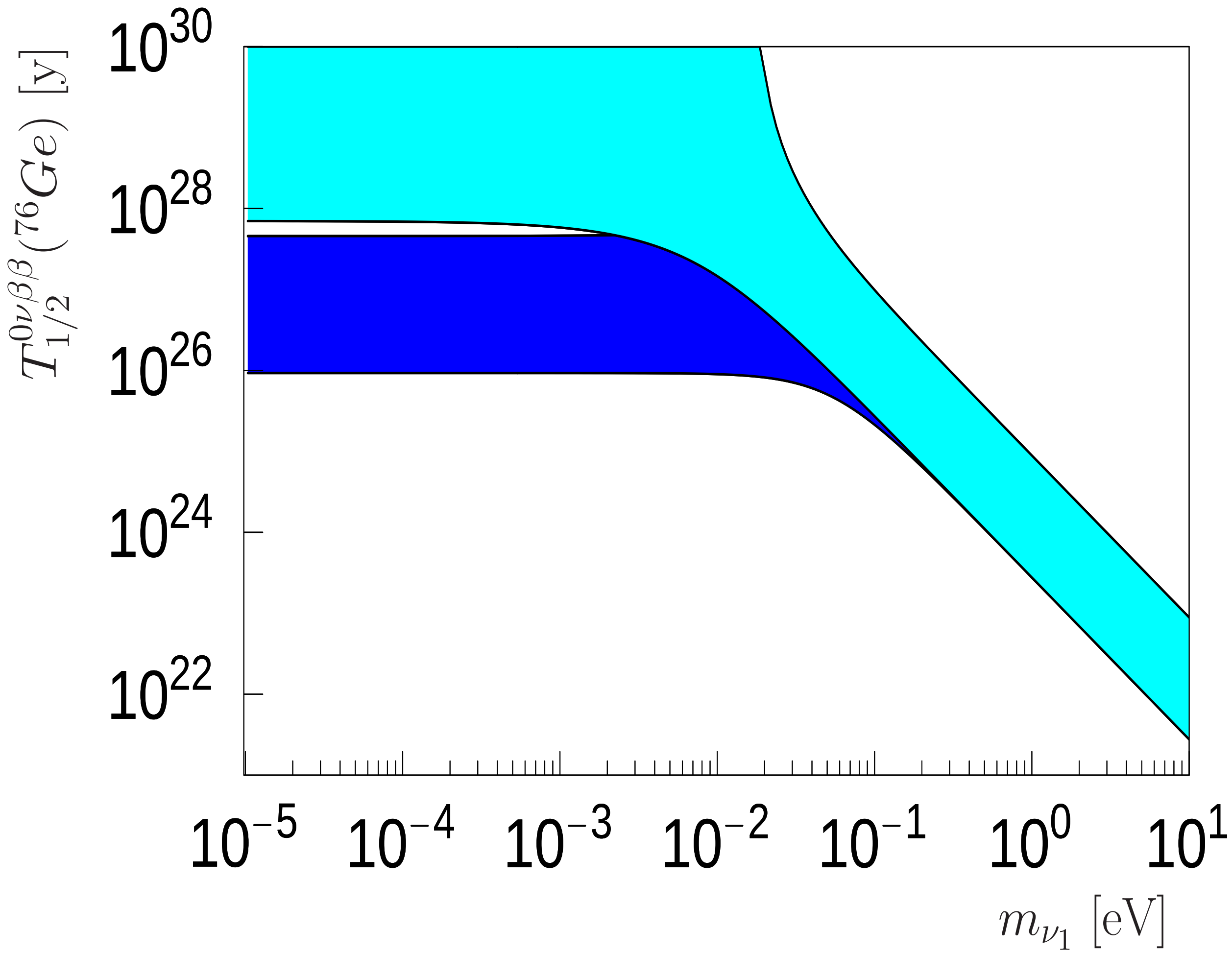
Normal + inverse hierarchy



$\Delta m^2_{Atm} = [1.4, 3.3] \cdot 10^{-3}$ eV^2, $\Delta m^2_{\odot} = [7.2, 9.1] \cdot 10^{-5}$ eV^2,

$\sin^2\theta_{\odot} = [0.23, 0.38]$, $\sin^2\theta_R = [0, 0.051]$

$T_{1/2}^{0\nu\beta\beta}$ and m_ν

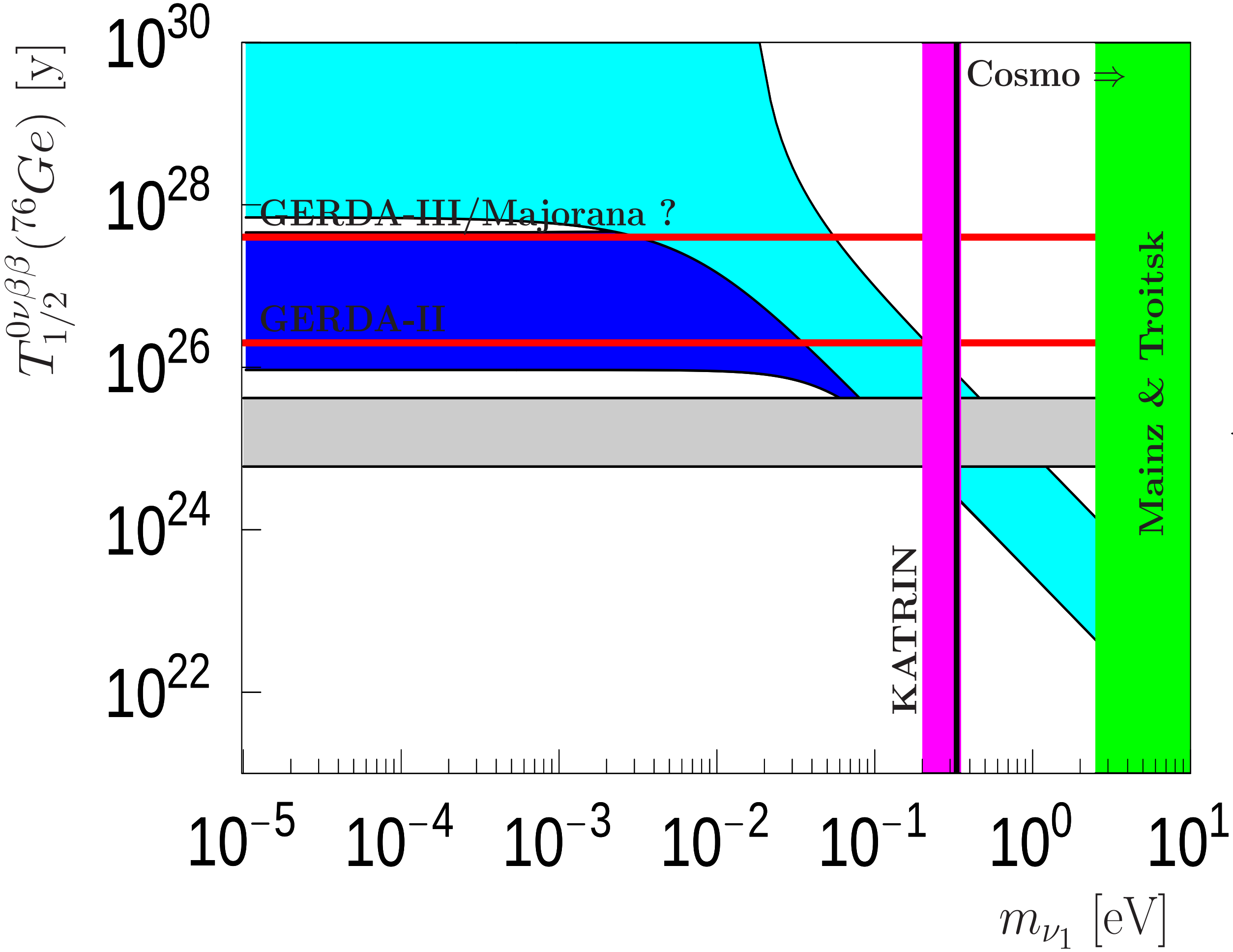


M_{m_ν} from K. Muto, PLB **391**, 243

$\Delta m^2_{Atm} = [1.4, 3.3] \cdot 10^{-3}$ eV^2, $\Delta m^2_\odot = [7.2, 9.1] \cdot 10^{-5}$ eV^2,

$\sin^2\theta_\odot = [0.23, 0.38]$, $\sin^2\theta_R = [0, 0.051]$

Current data: $0\nu\beta\beta$ and m_ν



⇐ "KDHK" claim
NIMA 522

$$\Delta m^2_{Atm} = [1.4, 3.3] \cdot 10^{-3}\ \text{eV}^2, \qquad \Delta m^2_\odot = [7.2, 9.1] \cdot 10^{-5}\ \text{eV}^2,$$

$$\sin^2\theta_\odot = [0.23, 0.38], \qquad \sin^2\theta_R = [0, 0.051]$$

$\mathcal{IV}.$

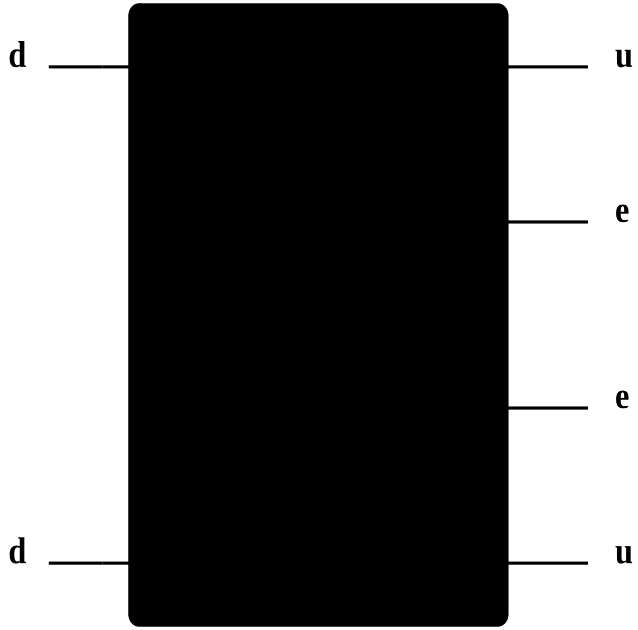
Lepton Number Violation and $0\nu\beta\beta$

Black Box I.

An ideal experiment detects appearance of two electrons:

$L = 0 \quad \Rightarrow \quad L = 2?$

Observables?

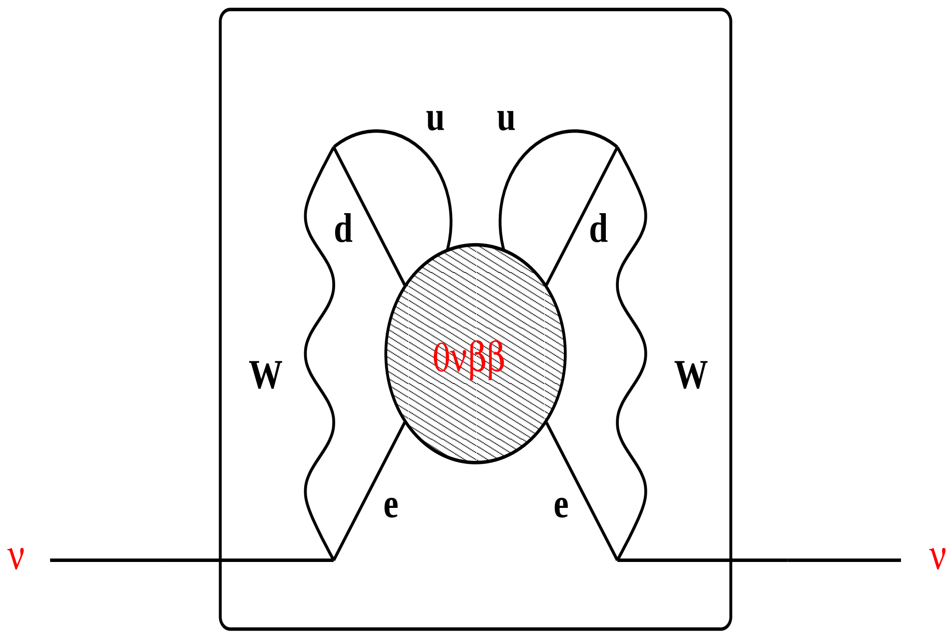


i) Sum energy $\Rightarrow 0\nu\beta\beta$

ii) $\#$ Events $\Rightarrow$ Half-life

iii) - ?) ... Others ... (?)

Black Box Theorem



Schechter & Valle, PRD 1982
Takasugi, PLB 1984

If $0\nu\beta\beta$
is observed
the neutrino is a
Majorana particle!

$\Rightarrow$ $\langle m_\nu \rangle$ must appear in higher order, even if suppressed or absent at tree-level

$\Rightarrow$ Qualitative statement only:
Value of m_ν depends on model

Again: Mass mechanism

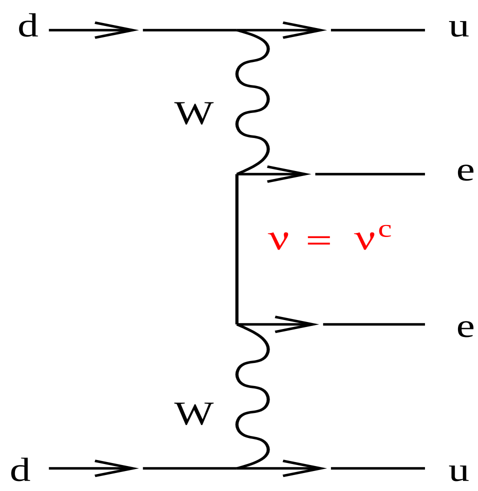
Standard model weak current at low energy:

$$\mathcal{L}^{\rm SM} = \frac{G_F}{\sqrt{2}} j^{\mu}_{V-A} J^{\dagger}_{V-A,\mu}$$

with $J^{\dagger}_{V-A,\mu} = \bar{u}\gamma_\mu P_L d$, $j^{\mu}_{V-A} = \bar{e}\gamma^\mu P_L \nu$

$\Rightarrow$ Product of two $\mathcal{L}$

$+$ Majorana neutrino mass:



Neutrino propagator:

$$P_L \int \frac{d^4p}{(2\pi)^4} \frac{m_\nu + \not{p}}{p^2 - m_\nu^2} P_L$$

$$= P_L \int \frac{d^4p}{(2\pi)^4} \frac{m_\nu}{p^2 - m_\nu^2}$$

$$\sim m_\nu$$

Lorentz-invariant Lagrangian

Consider all possible currents (V, A, S, P, T):

$$J^\dagger_\alpha = \bar{u}\mathcal{O}_\alpha d$$
$$j_\alpha = \bar{e}\mathcal{O}_\alpha \nu$$
$$j^{\text{short}}_\alpha = \bar{e}\mathcal{O}_\alpha e^C$$

Define operators $\mathcal{O}_{\alpha,\beta}$ with definite helicity:

$$\mathcal{O}_{V-A} = \gamma^\mu P_L \qquad \mathcal{O}_{V+A} = \gamma^\mu P_R$$
$$\mathcal{O}_{S-P} = P_L \qquad \mathcal{O}_{S+P} = P_R$$
$$\mathcal{O}_{T_L} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] P_L \qquad \mathcal{O}_{T_R} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] P_R$$

Lorentz-invariant Lagrangian

Write down most general $\mathcal{L}$:

$$\mathcal{L} = \mathcal{L}^{\rm SM} + \mathcal{L}^{\rm long} + \mathcal{L}^{\rm short}$$

Standard model:

$$\mathcal{L}^{\rm SM} = \frac{G_F}{\sqrt{2}} j^{\mu}_{V-A} J^{\dagger}_{V-A,\mu}$$

Long-range part:

$$\mathcal{L}^{\rm long} = \frac{G_F}{\sqrt{2}} \sum_{\alpha,\beta} \epsilon^{\beta}_{\alpha} j_\beta J^{\dagger}_{\alpha}$$

with:

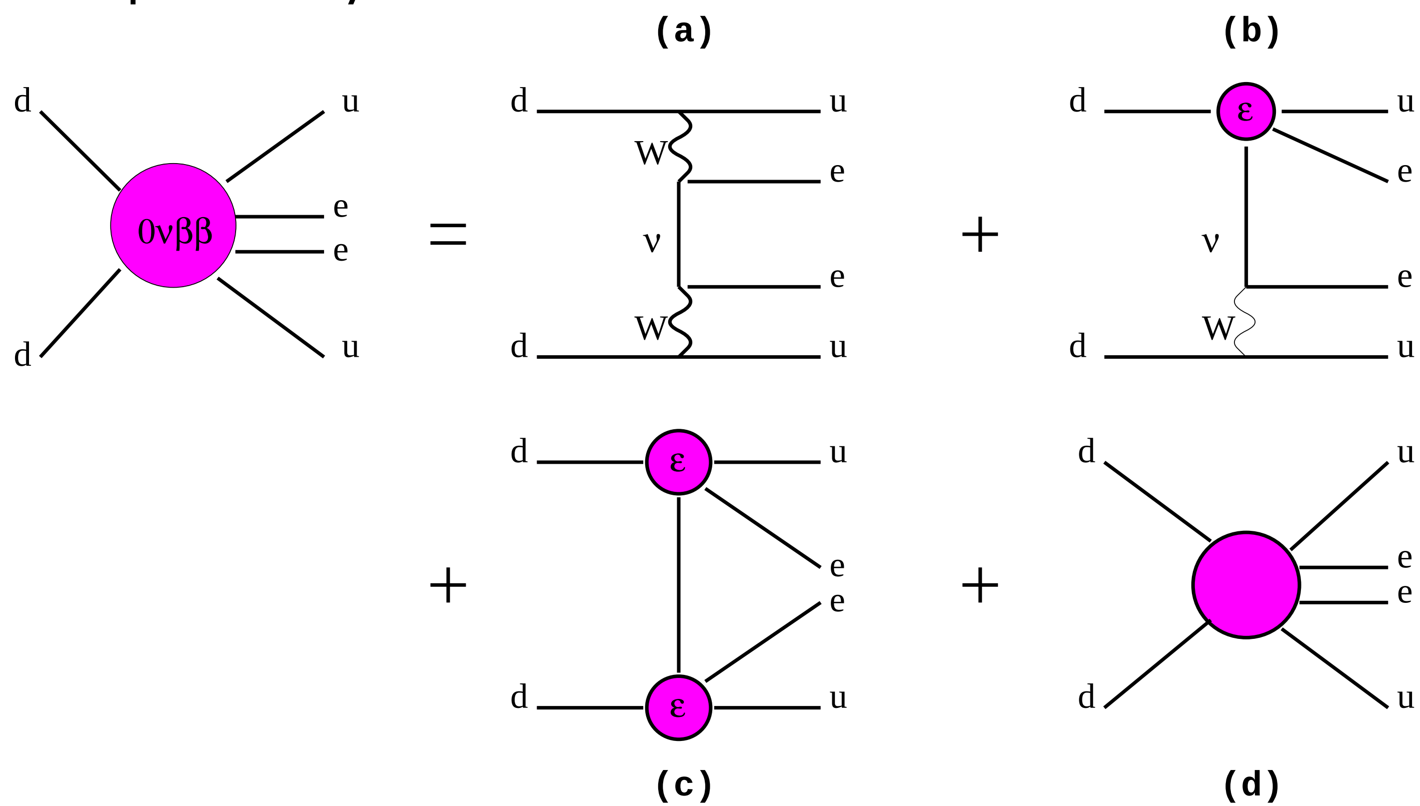
$$J^{\dagger}_{\alpha} = \bar{u}\mathcal{O}_\alpha d,$$
$$j_\beta = \bar{e}\mathcal{O}_\beta \nu$$
$$j^{\rm short}_{\beta} = \bar{e}\mathcal{O}_\beta e^C$$

Short-range part:

$$\mathcal{L}^{\rm short} = \frac{G_F^2}{2m_P} \sum_{\alpha,\beta\gamma} \epsilon^{\alpha\beta\gamma} j^{\rm short}_{\alpha} J^{\dagger}_{\beta} J^{\dagger}_{\gamma}$$

Lorentz-invariant Lagrangian

Graphically:



⇒ Neglect terms proportional ϵ^2

⇒ For limits, consider only helicity enhanced terms, i.e. $\sim \not{p}$

Neutrino propagator:

$$P_L \int \frac{d^4p}{(2\pi)^4} \frac{m_\nu + \not{p}}{p^2 - m_\nu^2} P_R$$

$$= P_L \int \frac{d^4p}{(2\pi)^4} \frac{\not{p}}{p^2 - m_\nu^2}$$

Limits: $T_{1/2}(^{76}Ge) \geq 1.2 \cdot 10^{25}$ *ys*

H. Päs et al., PLB 1999 and 2001:

Long range part: Short range part:

ϵ^{V+A}_{V-A}	$4.3 \cdot 10^{-9}$		$\epsilon^{S/P,S/P,S/P}$	$4 \cdot 10^{-7}$
ϵ^{V+A}_{V+A}	$7.9 \cdot 10^{-7}$		$\epsilon^{T,T,S/P}$	$2.5 \cdot 10^{-9}$
ϵ^{S+P}_{S-P}	$1.1 \cdot 10^{-8}$		$\epsilon^{V\pm A,V\pm A,S/P}$	$5 \cdot 10^{-8}$
ϵ^{S+P}_{S+P}	$1.1 \cdot 10^{-8}$		$\epsilon^{V\pm A,V\mp A,S/P}$	$1.4 \cdot 10^{-8}$
ϵ^{TR}_{TL}	$6.4 \cdot 10^{-10}$		$\epsilon^{V/A,T,V/A}$	$2.5 \cdot 10^{-8}$
ϵ^{TR}_{TR}	$1.7 \cdot 10^{-9}$		$\epsilon^{V/A,S/P,V/A}$	$2.5 \cdot 10^{-7}$

$\Rightarrow$ Limits are "on-axis", i.e. one ϵ non-zero

Limits on

physics beyond the SM

Left-right symmetric models

Translation table:

Päs et al.	DKT		limit
ϵ^{V+A}_{V-A}	$\langle\eta\rangle$	$\sim \sum_j U_{ej} V_{ej} \tan\zeta$	$4.3 \cdot 10^{-9}$
ϵ^{V+A}_{V+A}	$\langle\lambda\rangle$	$\sim \sum_j U_{ej} V_{ej} (m_{W_L}/m_{W_R})^2$	$7.9 \cdot 10^{-7}$
$\epsilon^{V\pm A, V\mp A, S/P}$	$\langle\xi\rangle^*$	$\sim \sum_j V_{ej}^2 / m_N (m_{W_L}/m_{W_R})^2$	$1.4 \cdot 10^{-8}$

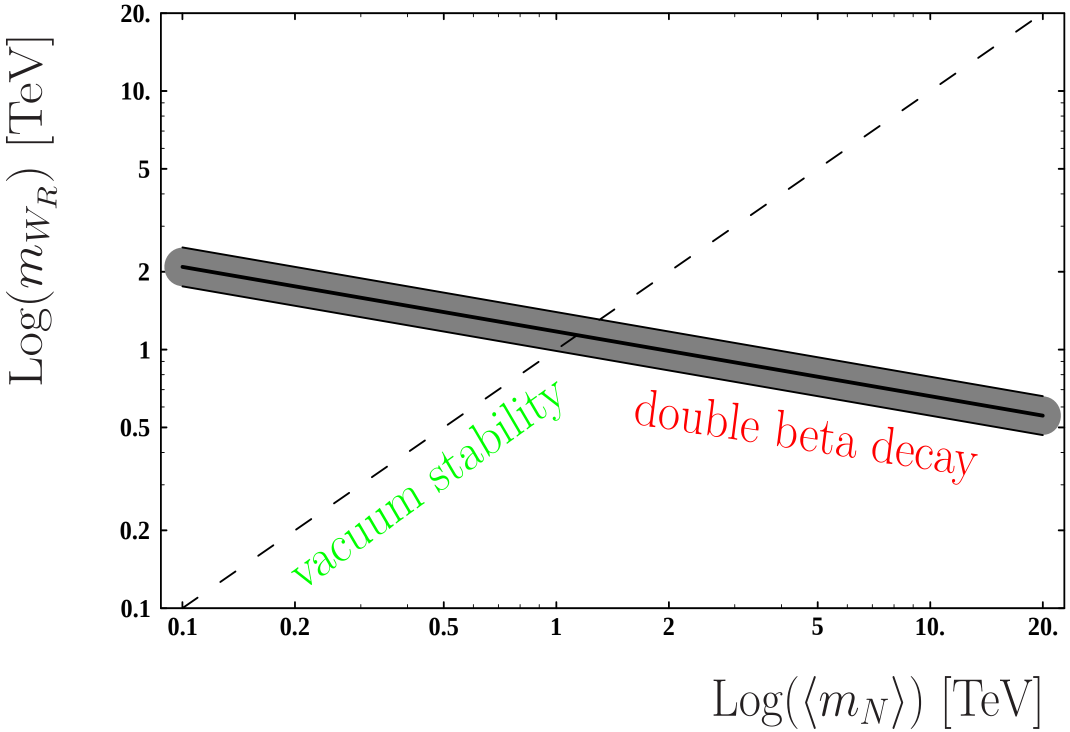
* Notation of Hirsch et al., PLB 1996

Note:

$\Rightarrow$ $\langle\eta\rangle$ and $\langle\lambda\rangle$ are long-range

$\Rightarrow$ $\langle\xi\rangle$ is short-range

Limit on m_{W_R} from $0\nu\beta\beta$



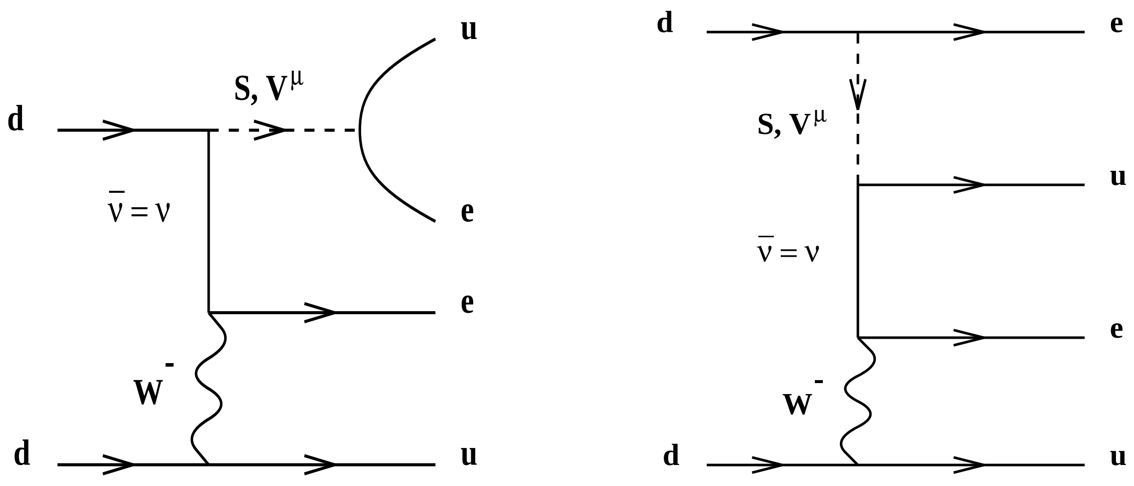
Mohapatra, PRD 1986

Hirsch et al., PLB 1996

Note: Width of band indicates NME uncertainty of ~ 2

$$\langle \xi \rangle \Rightarrow \quad m_{W_R} \gtrsim 1.3 \left(\frac{\langle m_N \rangle}{[1\mathrm{TeV}]} \right)^{-1/4} \mathrm{TeV}$$

Leptoquarks



LQs are hypothetical particles, coupling to a quark-lepton pair

LQs can be scalars (S) or vectors (V^μ)

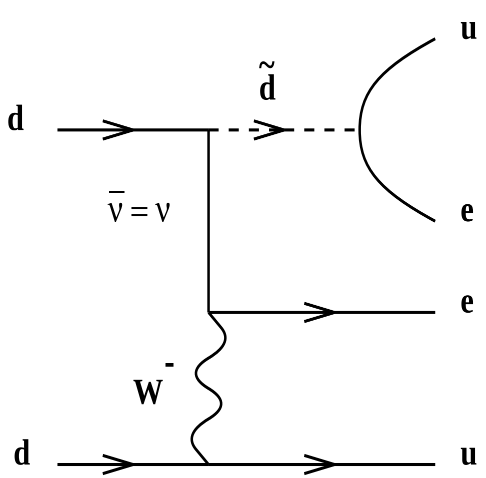
LQs can violate L (in LQ-Higgs interaction) if conserve B

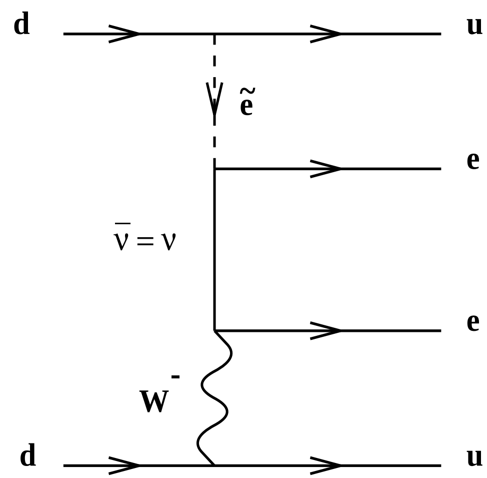
$$
\begin{aligned}
\mathcal{L}_{LQ}^{eff} &= (\bar{\nu}_n P_R e^c) \left[\frac{\epsilon_S}{M_S^2} (\bar{u} P_R d) + \frac{\epsilon_V}{M_V^2} (\bar{u} P_L d) \right] - \\
&- (\bar{\nu}_n \gamma^\mu P_L e^c) \\
&\times \left[\left(\frac{\alpha_S^{(R)}}{M_S^2} + \frac{\alpha_V^{(R)}}{M_V^2} \right) (\bar{u} \gamma_\mu P_R d) - \sqrt{2} \left(\frac{\alpha_S^{(L)}}{M_S^2} + \frac{\alpha_V^{(L)}}{M_V^2} \right) (\bar{u} \gamma_\mu P_L d) \right],
\end{aligned}
$$

$\Rightarrow$ ϵ and α products of LQ couplings times mixing angle

$\Rightarrow$ Example: $\alpha_{S,V}^{(L)} \leq 2.5 \cdot 10^{-10} \left(\frac{M_{S;V}}{100 \mathrm{GeV}} \right)^2$ Hirsch et al. PRD 1996

Long-range RPV SUSY





Babu & Mohapatra, PRL 1995

Päs et al., PLB 1999

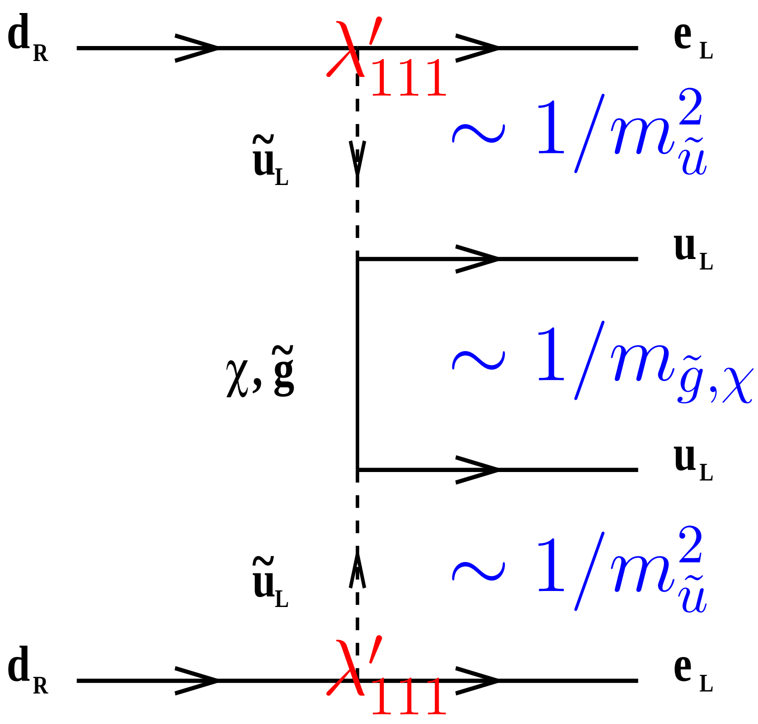
$\Rightarrow T(^{76}Ge) \geq 1.2 \cdot 10^{25}$ ys

$\Rightarrow$ Including tensor matrix element:

$$\lambda'_{113}\lambda'_{131} \leq 3.8 \cdot 10^{-8} \left(\frac{m_{SUSY}}{100\text{GeV}}\right)^3$$

Short-range RPV SUSY

Only one example:



Mohapatra, PRD 1986
Vergados, PLB 1987
Hirsch et al., PRL 1995

Amplitude
$\sim$ 2 RPV vertices,
but limit
very stringent

From gluino graph:

$$\lambda'_{111} \leq 3 \times 10^{-4} \left(\frac{m_{\tilde{q}}}{100\mathrm{GeV}}\right)^2 \left(\frac{m_g}{100\mathrm{GeV}}\right)^{1/2}$$

$\langle m_\nu \rangle$ or BSM?

Angular correlation

Calculate differential width:

For LR-models:
Doi, Kotani & Takasugi, 1985

$$\frac{d\Gamma}{d\cos\theta} \sim (1 - K\cos\theta)$$

For general LI Lagrangian:
Ali, Borisov & Zhuridov, 2007

$\Rightarrow$ Advantage: K depends strongly on mechanism, but weakly on nuclear matrix elements (i.e. weakly on isotope)

$\Rightarrow$ Disadvantage: Many terms in general Lagrangian lead to same (or very similar) angular dependence

$\Rightarrow$ Disadvantage: Most experiments calorimetric measurements only, exception: NEMO-III

Double beta plus decays

In $\beta^+\beta^+$ three principle decay modes:

For LR-models: Hirsch et al., Z. Phys. A 1994

For general LI Lagrangian: no publication exists

$0\nu\beta^+\beta^+$: $(Z,N) \Rightarrow (Z-2,N) + 2e^+$

$0\nu\beta^+/EC$: $(Z,N) + e^- \Rightarrow (Z-2,N) + e^+$

$0\nu EC/EC$: $(Z,N) + 2e^- \Rightarrow (Z-2,N)^*$

Numerical example: ^{124}Xe using pn-QRPA

Mode:	C_{mm}	$C_{\lambda\lambda}$	$C_{\lambda\lambda}/C_{mm}$
$0\nu\beta^+\beta^+$	$8.7 \cdot 10^{-17}$	$8.5 \cdot 10^{-18}$	0.098
$0\nu\beta^+/EC$	$1.6 \cdot 10^{-15}$	$2.2 \cdot 10^{-14}$	13.75

$\Rightarrow$ Advantage: Ratios (nearly) independent from nuclear matrix elements uncertainty

$\Rightarrow$ Disadvantage: Only $\langle\lambda\rangle$ enhanced

$\Rightarrow$ Disadvantage: Even best isotopes (at least) one order of magnitude slower than best $\beta^-\beta^-$

Others?

$\Rightarrow$ Compare rates ground state to 2^+

- Doi, Kotani & Takasugi, 1985: YES
- Tomoda, PLB 2000: NO for $\langle\eta\rangle$, maybe for $\langle\lambda\rangle$
- Disadvantage: several o.m. slower than g.s. transitions

$\Rightarrow$ Compare rates ground state to 0_1^+

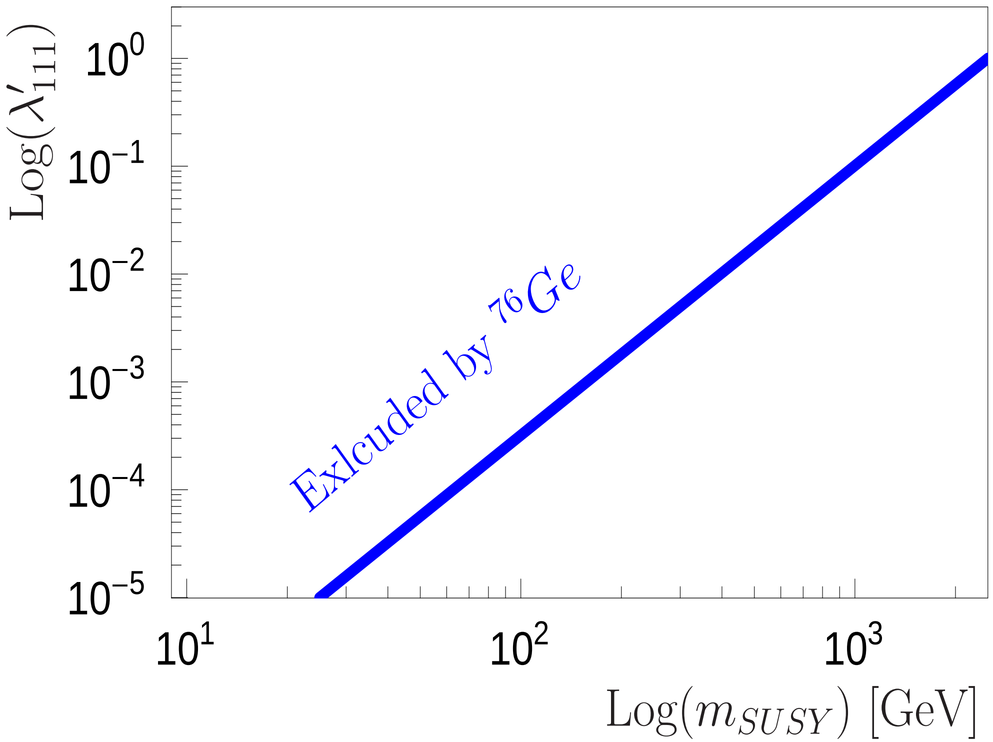
- Simkovic & Fäßler, Prog. Part. Nucl. Phys, 2002
- Disadvantage: (a) (about) 2 o.m. slower than g.s. transitions
- Disadvantage: (b) need to know matrix elements: $\Delta M \ll 40\%$

$\Rightarrow$ Compare rates, different nuclei

- Päs & Deppisch, PRL 2007; Gehman & Elliott, J. Phys. G 2007
- Disadvantage: need to know matrix elements: $\Delta M \ll x$,
  x differs for different particle physics, but is
  $x = (10 - 25)\%$, depending on number of isotopes

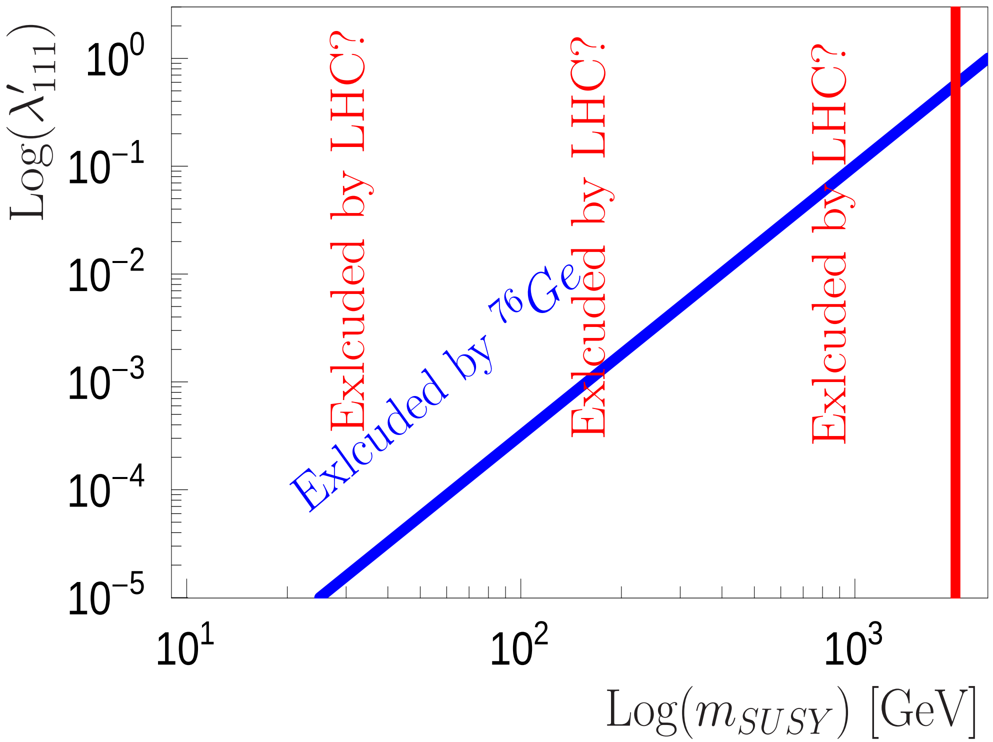
From accelerators?

In trilinear RPV SUSY, limit from ^{76}Ge excludes:



From accelerators?

In trilinear RPV SUSY, limit from ^{76}Ge excludes:



No SUSY @ LHC?

RPV $0\nu\beta\beta$ decay (nearly) excluded

$\Rightarrow$ Disadvantage: RPV SUSY yes, but other models?

Conclusions

$\Rightarrow$ All models with lepton number violation contribute to $0\nu\beta\beta$ decay

$\Rightarrow$ Discovery of $0\nu\beta\beta$ decay $\equiv$ Majorana neutrinos

$\Rightarrow$ Current limits order $\mathcal{O}(1/2)$ eV, see S. Schönert's talk for experimental prospects

$\Rightarrow$ IF $0\nu\beta\beta$ decay is discovered: Which is the dominant mechanism?