CP, T and CPT Violation at Future Long Baseline Experiments

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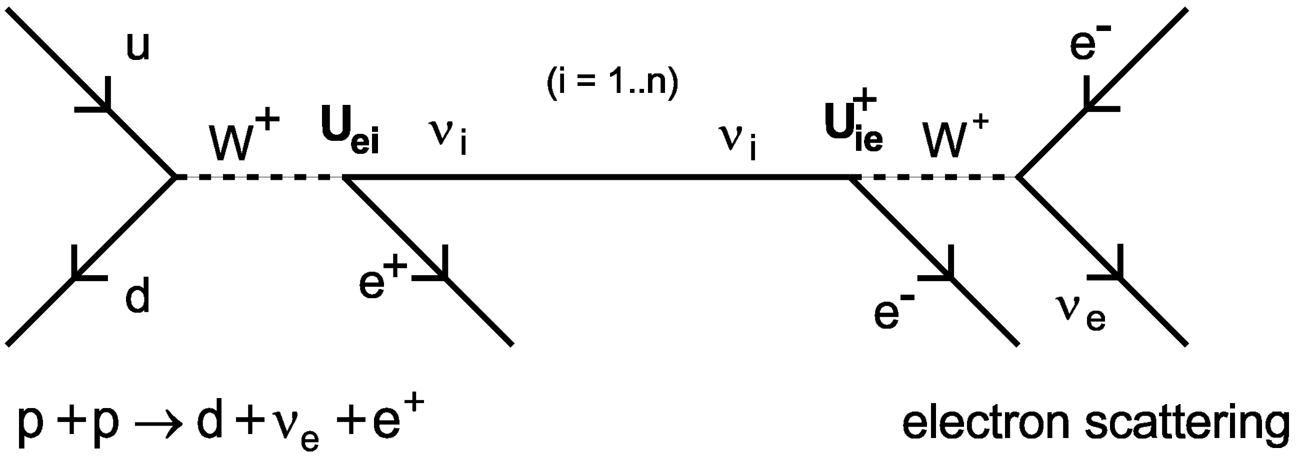
Neutrino Oscillations for $N = 2$

Production
Propagation
Detection



2 Flavours ν_e, ν_μ:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

flavour states = mixing matrix · mass eigenstates

Probability:

$$P(\nu_\mu \to \nu_e) = |\langle \nu_\mu(t) | \nu_e(t=0) \rangle|^2 = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

$N \geq 2$ with CP and Matter Effects

Precision: $N = 2$ description insufficient $\Rightarrow$ modifications

- $2 \rightarrow 3$ neutrino framework $\Rightarrow$ more parameters & CP effects
- MSW: parameter mapping in matter

$$\Rightarrow \quad P(\nu_{e_l} \rightarrow \nu_{e_m}) = \underbrace{\delta_{lm} - 4 \sum_{i>j} \mathrm{Re} J_{ij}^{e_l e_m} \sin^2 \Delta_{ij}}_{P_{CP}} \underbrace{-2 \sum_{i>j} \mathrm{Im} J_{ij}^{e_l e_m} \sin 2\Delta_{ij}}_{P_{\not{C}\not{P}}}$$

Shorthands: $J_{ij}^{e_l e_m} := U_{li} U_{lj}^* U_{mi}^* U_{mj} \qquad \Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E}$

Neutrinos: $P(\nu_{e_l} \rightarrow \nu_{e_m}) = P_{CP} + P_{\not{C}\not{P}}$

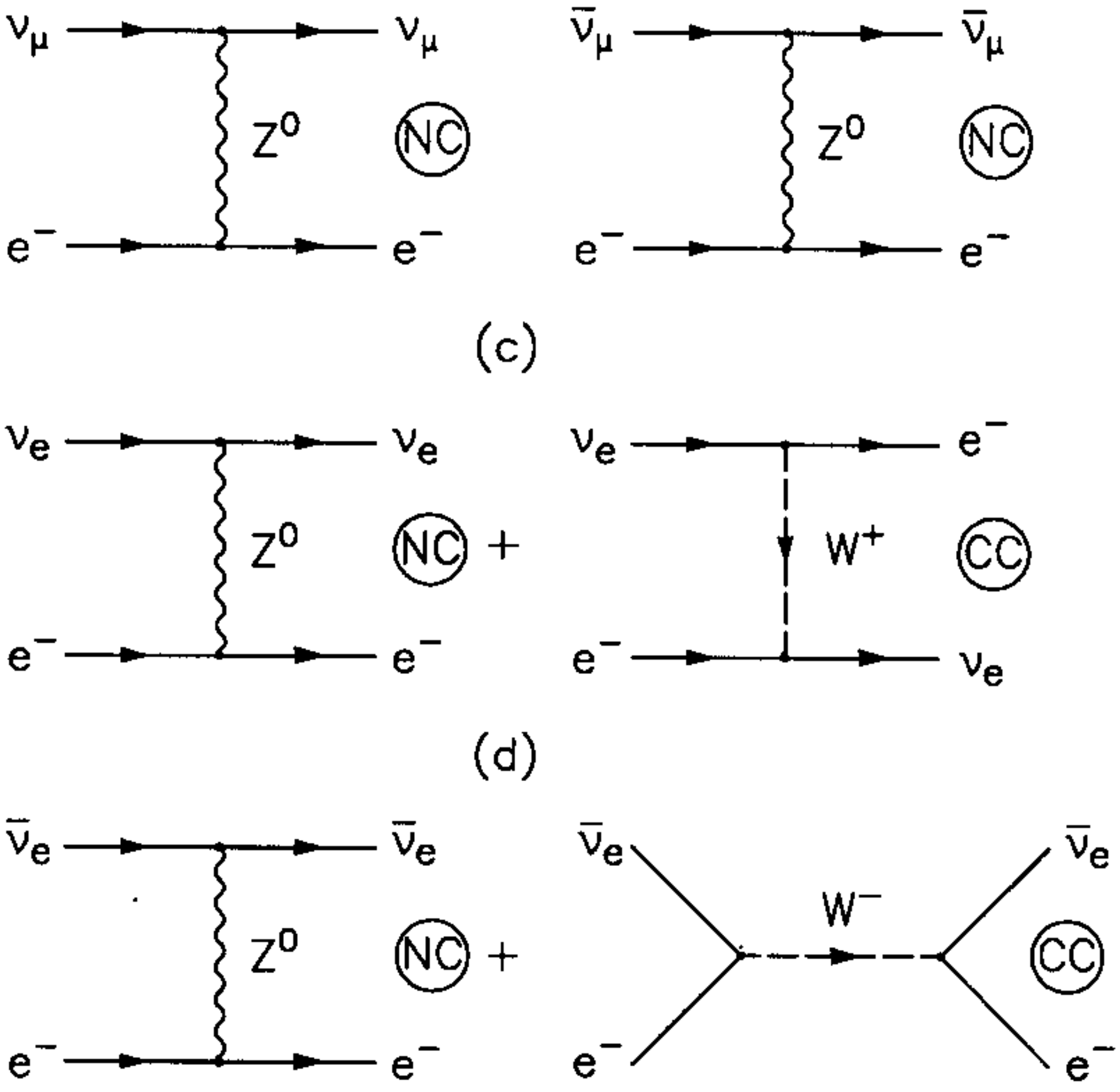
Antineutrinos: $P(\overline{\nu}_{e_l} \rightarrow \overline{\nu}_{e_m}) = P_{CP} - P_{\not{C}\not{P}}$

$\Rightarrow$ **e.g. CP Asymmetries:**

$$\mathbf{a}^{\mathrm{CP}} := \frac{\mathrm{P}(\nu_{e_l} \rightarrow \nu_{e_m}) - \mathrm{P}(\overline{\nu}_{e_l} \rightarrow \overline{\nu}_{e_m})}{\mathrm{P}(\nu_{e_l} \rightarrow \nu_{e_m}) + \mathrm{P}(\overline{\nu}_{e_l} \rightarrow \overline{\nu}_{e_m})} = \frac{P_{\not{C}\not{P}}}{P_{CP}}$$

Matter Effects and MSW Resonance

Mikheyev-Smirnov-Wolfenstein: coherent forward scattering



$\mathcal{L}_{NC}$ = **flavour universal**

$\mathcal{L}_{CC} = \sqrt{2} G_F n_e \quad \Leftrightarrow \quad$ **only** ν_e

MSW-resonance energy(Δm^2_{31})

Earth: $\mathbf{E_{res} \simeq 10\ GeV}$

dominated by average density

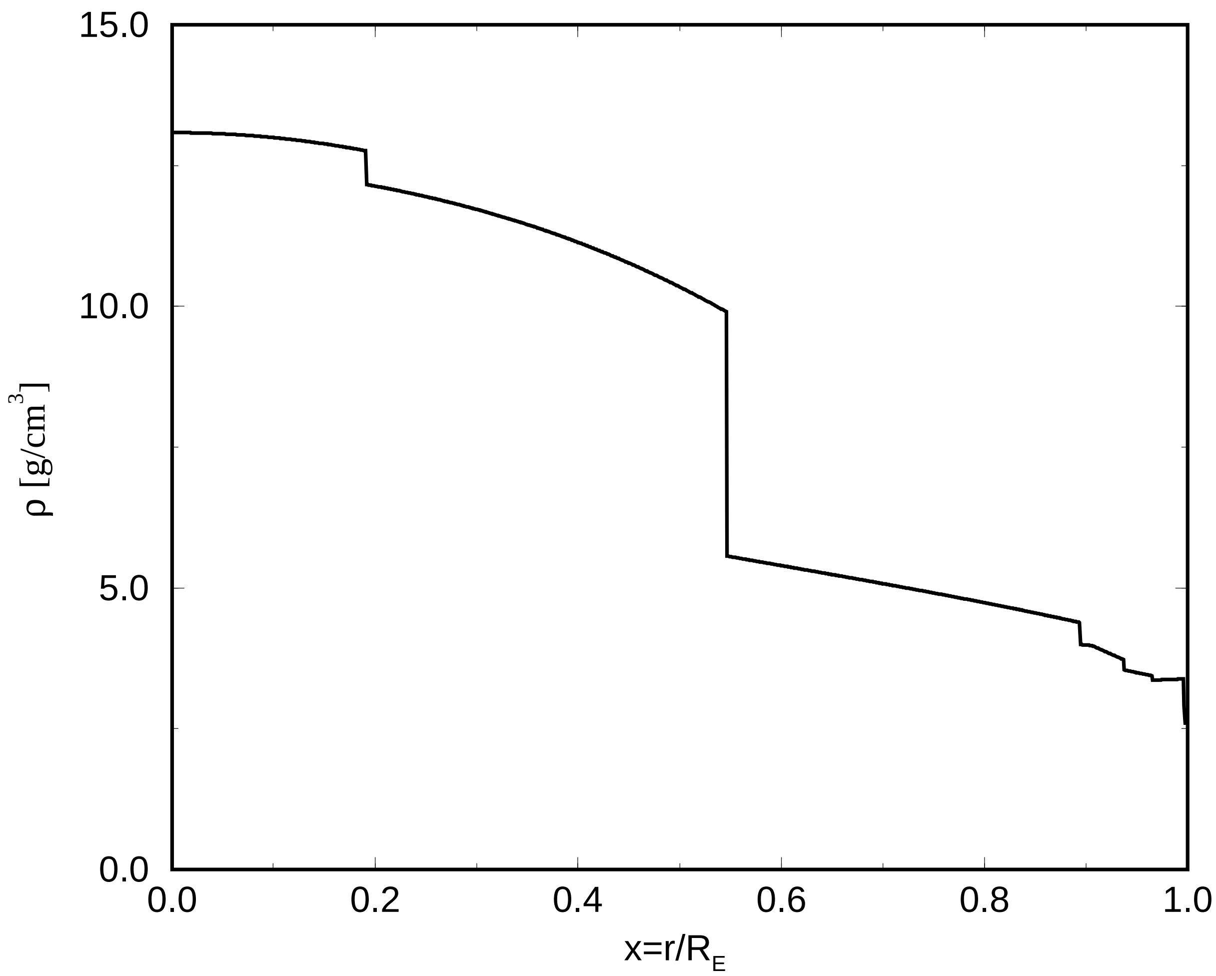
$$\rho = \rho_{\text{average}} + \delta\rho$$

Baseline & MSW Matter Effects

Beams in Earth Matter:

$\Rightarrow$ electron density profile
as function of radius Stacy
density errors Geller & Hara

Large $L \Rightarrow$ steep angles
$L =$ 2800 **km** $\Leftrightarrow$ 13^o
$L =$ 7300 **km** $\Leftrightarrow$ 35^o
$L =$ 12750 **km** $\Leftrightarrow$ 90^o
$L \leq \mathcal{O}(10\ 000\ \mathrm{km})$ $\Leftrightarrow$ mantle



- $E_{resonance} \simeq 10 - 15$ **GeV**, matter effects grow with distance L
- Average density profile uncertainties decrease with L $\Rightarrow$ $\simeq 5\%$ error

$\Delta m^2_{12} \simeq 0$, $Y = e^-$/nucleon ρ =matter density m_n =nucleon mass

In Flavour Basis:

$$H_0 + \delta \mathbf{H}_{CC} + \delta \mathbf{H}_{NC} = \frac{1}{2E}\mathbf{U}\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}\mathbf{U^T} + \frac{1}{2E}\begin{pmatrix} \mathbf{A}+\mathbf{A}' & 0 & 0 \\ 0 & \mathbf{A}' & 0 \\ 0 & 0 & \mathbf{A}' \end{pmatrix}$$

$$\mathbf{A} = \pm \frac{\mathbf{2\sqrt{2}G_F Y \rho E}}{\mathbf{m_n}},$$

"$+$" for $\nu \oplus$ matter **and** $\overline{\nu} \oplus \overline{\text{matter}}$

$\mathbf{U} = R_{23}R_{13}R_{12} \rightarrow \mathbf{U}' \Rightarrow$ **1-3 subspace parameter mapping**

$$\begin{aligned} \sin^2 2\theta'_{13} &= \frac{\sin^2 2\theta_{13}}{C_\pm^2} \\ \Delta m^2_{31,m} &= \Delta m^2 C_\pm \\ \Delta m^2_{32,m} &= \frac{\Delta m^2\ (C_\pm + 1) + A}{2} \\ \Delta m^2_{21,m} &= \frac{\Delta m^2\ (C_\pm - 1) - A}{2} \end{aligned}$$

$$C_\pm^2 = \left(\frac{A}{\Delta m^2} - \cos 2\theta\right)^2 + \sin^2 2\theta$$

$\Rightarrow$ **Different mappings for neutrinos and antineutrinos**

⇒ **All CP-violating effects are proportional to:**

1) $8J_{\text{CP}} = \cos\theta_{13}\sin(2\theta_{13})\sin(2\theta_{12})\sin(2\theta_{23})\sin\delta$

2) $\sin\Delta_{12}\sin\Delta_{23}\sin\Delta_{31}$ ⇒ **3 different masses required**

$P_{\not{C}\not{P}}$ is suppressed by:

- **small solar Δm^2_{sun}:** $\sin\Delta_{12} \approx \Delta_{12} \ll 1$
- **small mixing angles:** $\sin^2 2\theta_{13} \ll 1$
- **in addition for SMA:** $\theta_{\text{sun}} \simeq \theta_{12}$

P_{CP} is not suppressed by $\Delta_{12} = \frac{\Delta m^2_{12}\,L}{4E}$

Sizable CP effects:

3 Neutrinos ⇒ LMA-MSW (most likely)
4 Neutrinos ⇒ always

Qualitative Analytic Understanding

- **full numerical simulation**
- $\Delta = \Delta m^2_{31} L/4E$
- **qualitative understanding** $\Rightarrow$ **expand in** $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ **and** $\sin^2 2\theta_{13}$
- **matter effects** $\hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31}; \;\; V = \sqrt{2}G_F n_e$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \cos^2\theta_{13}\sin^2 2\theta_{23}\sin^2\Delta + 2\,\alpha\;\cos^2\theta_{13}\cos^2\theta_{12}\sin^2 2\theta_{23}\Delta\cos\Delta$$

$$\begin{aligned}
P(\nu_e \to \nu_\mu) \approx\; & \sin^2 2\theta_{13}\sin^2\theta_{23}\,\frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
& \pm\; \sin\delta_{\mathrm{CP}}\;\alpha\sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin(\Delta)\frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
& +\; \cos\delta_{\mathrm{CP}}\,\alpha\sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\cos(\Delta)\frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
& +\; \alpha^2\sin^2 2\theta_{12}\cos^2\theta_{23}\frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\end{aligned}$$

Freund

Cervera & Donini & Gavela & Gomez Cadenas & Hernandez & Mena & Rigolin

Freund & Huber & ML

The Importance of θ_{13}

CHOOZ $\Rightarrow$ θ_{13} **is small:** $\sin^2 2\theta_{13} < 0.1$

All effects in the $\nu_e \to \nu_\mu$-transition depend crucially on θ_{13} :

- **the total transition rate**
- **matter effects**
- **the effects due to the sign of Δm^2_{31}**
- **CP violating effects**

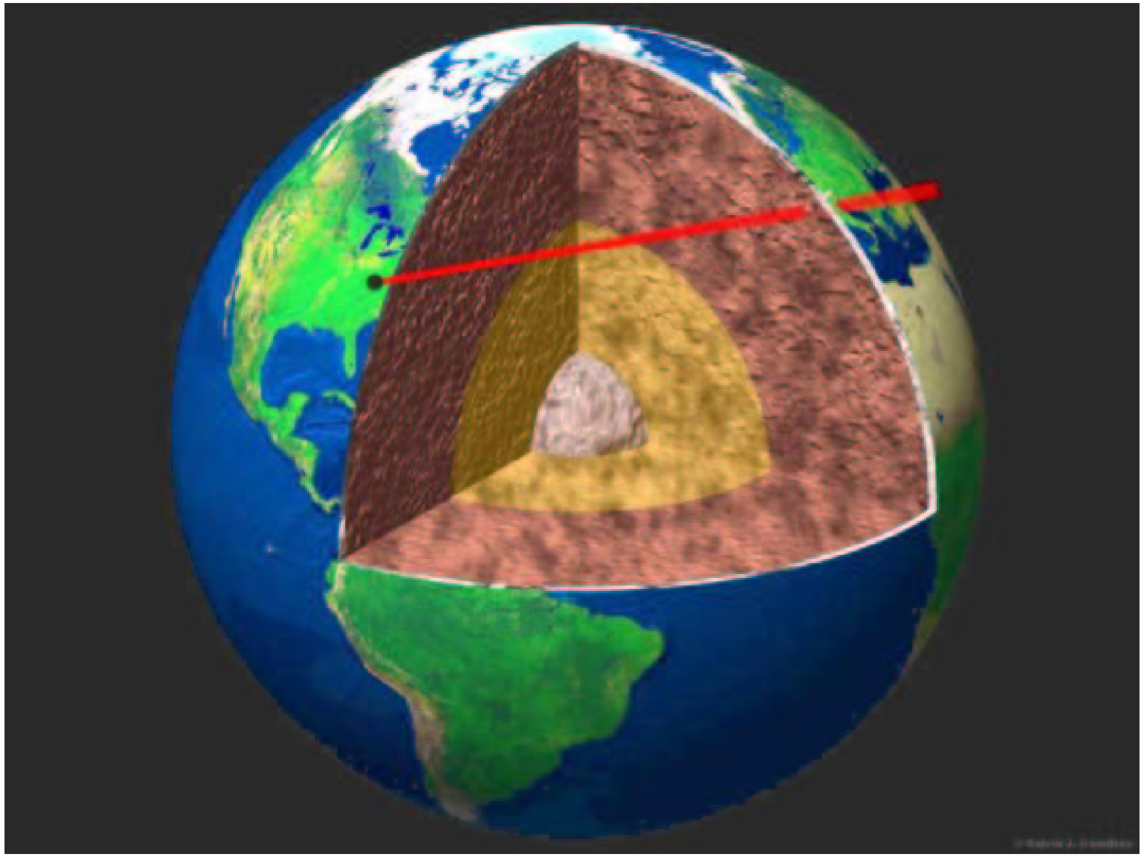
The size of θ_{13} determines if these effects can be studied

Errors, Degeneracies and Correlations

Look at expansion in powers of $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ **and** $\sin\theta_{13}$; $\Delta = \Delta m^2_{31} L/4E$; $V = 0$

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) \quad &\approx \quad \sin^2 2\theta_{13} \sin^2\theta_{23} \quad \sin^2(\Delta) \\
&\pm \quad \sin\delta_{\mathrm{CP}} \; \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin^3(\Delta) \\
&+ \quad \cos\delta_{\mathrm{CP}} \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \sin^2(\Delta) \\
&+ \quad \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \sin^2(\Delta)
\end{aligned}
$$

- **depends on solar parameters only via the product** $\Delta m^2_{21} \cdot \sin 2\theta_{12}$
- **most interesting:** $\sin^2 2\theta_{13} \simeq \alpha \cdot \sin^2 2\theta_{12} = \sin^2 2\theta_{12} \Delta m^2_{21}/\Delta m^2_{31}$
- **degeneracies**: **(**$\delta_{CP} - \theta_{13}$**), (**$\delta_{CP} - sign(\Delta m^2_{31})$**), (**$\theta_{23} - \frac{\pi}{2} - \theta_{23}$**)**
- $\alpha^2 = (\Delta m^2_{21})^2/(\Delta m^2_{31})^2$ **term dominates for tiny** $\sin^2 2\theta_{13}$ $\Leftrightarrow$ **error of** Δm^2_{21}



Precision!

Source $\otimes$ **Oscillation** $\otimes$ **Detector**

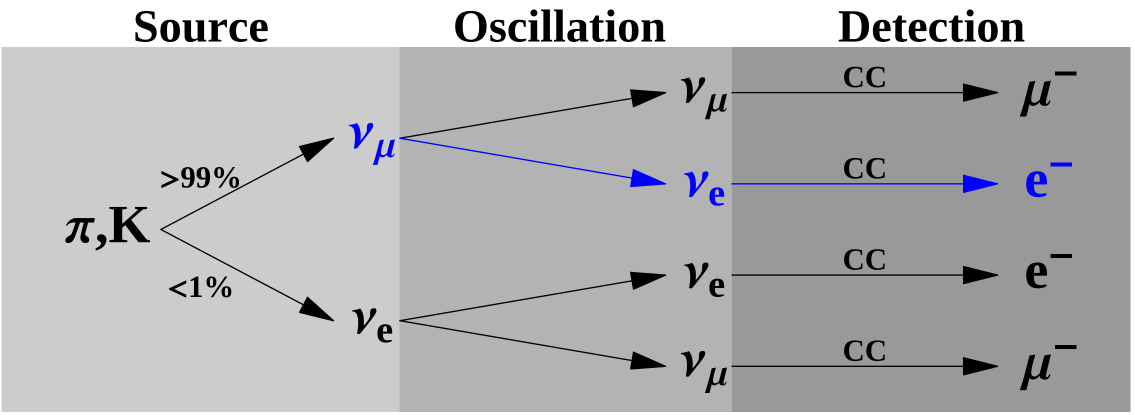
Source
- neutrino energy E
- flux and spectrum
- flavour composition
- contamination
- symmetric $\nu/\overline{\nu}$ operation

Oscillation
- oscillation channels
- realistic baselines
- MSW matter profile

Detector
- effective mass, material
- threshold, resolution
- particle ID (flavour, charge, event reconstruction, ...)
- backgrounds
- x-sections (at low E)

Sources, Oscillation and Detection

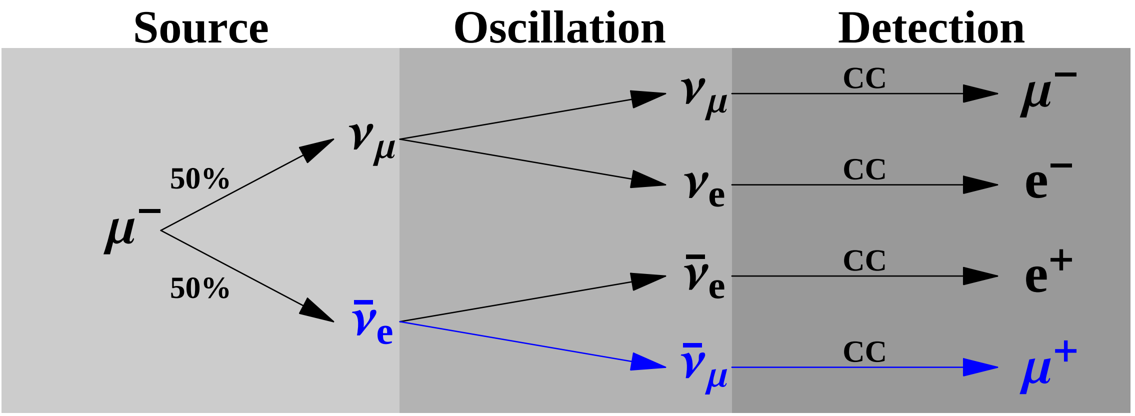
A) Conventional ν-Beams from Beam Dumps $\Rightarrow$ Superbeams



$\nu_\mu \to \nu_e$ oscillation most interesting
ν_e contamination $\Leftrightarrow$ off-axis
good electron detection efficiency
good NC background rejection
near detector
$\overline{\nu}$-beam $\simeq$ different experiment

B) Neutrino Factories

Geer



$\overline{\nu}_e \to \overline{\nu}_\mu$ oscillation most interesting
excellent beam properties
very good charge ID required
good NC background rejection
μ^+ mode very symmetric

C) Radioactive Beams ?

Pure ν_e or $\overline{\nu}_e$ beam from β decay, $\gamma \simeq 100$ Zuchelli

The Potential of Future LBL Setups

Physis input:

- full numerical 3-ν simulation and analytic understanding
- matter density profile and errors taken into account
- error correlations included (no fixed unknown parameters)

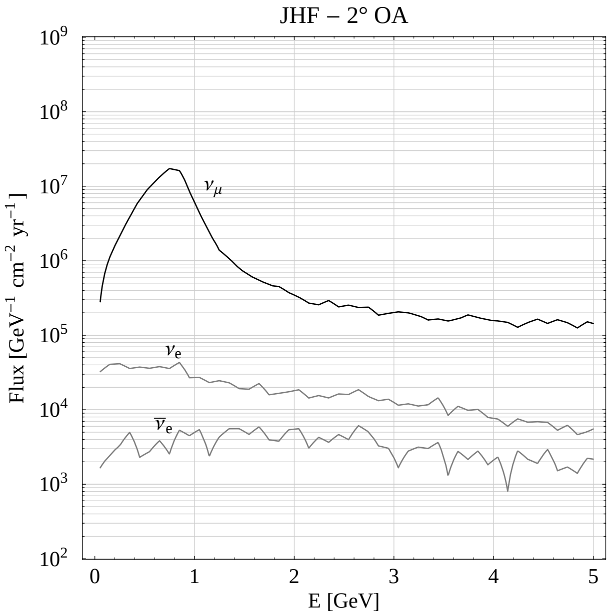
Analysis:

- MC for all possible input parameters
- combined fit of appearance and disappearance channels for both polarities
- fit spectral information and/or total rates
- adequate statistical methods (small event rates/bin):
  - $\Rightarrow$ Poissonian statistics
  - $\Rightarrow$ parameterization of systematical uncertainties
  - $\Rightarrow$ integrating out the nuisance parameters
  - $\Rightarrow$ projection on the parameter of interest
  - $\Rightarrow$ external information from KamLand (=LMA) and geophysics
  - $\Rightarrow$ 6 remaining parameters

$\Rightarrow$ Extract parameters and (correlated) errors $\Leftrightarrow$ sensitivity

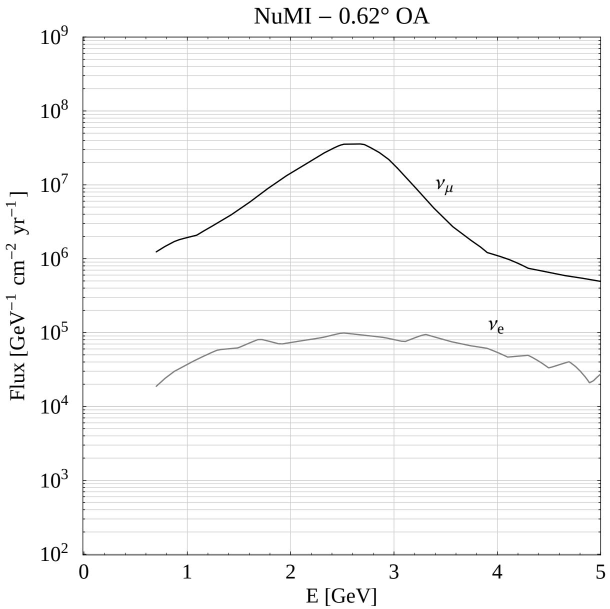
The Sources

JHF



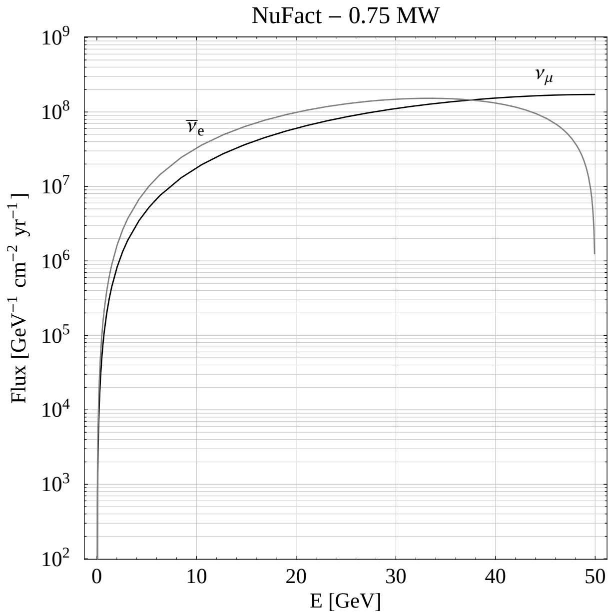
- **mean energy** $0.51\,\text{GeV}$
- **peak intensity** $1.7 \cdot 10^7\,\text{GeV}^{-1}\,\text{cm}^{-2}\,\text{yr}^{-1}$ **at** $0.78\,\text{GeV}$
- ν_μ/ν_e**-ratio at peak** 0.2%

NuMI off-axis



- **mean energy** $2.78\,\text{GeV}$
- **peak intensity** $3.6 \cdot 10^7\,\text{GeV}^{-1}\,\text{cm}^{-2}\,\text{yr}^{-1}$ **at** $2.18\,\text{GeV}$
- ν_μ/ν_e**-ratio at peak** 0.2%

NuFact



- **mean energy** $30\,\text{GeV}$
- **peak intensity** $1.5 \cdot 10^8\,\text{GeV}^{-1}\,\text{cm}^{-2}\,\text{yr}^{-1}$ **at** $33.33\,\text{GeV}$
- ν_μ/ν_e**-ratio at peak** 83%

Uncertainties in flux and ν_e–background

Uncertainties in flux

Itow et al.

Para, Szleper

Geer

Detectors

	water Cherenkov = SK (HK)	low-Z calorimeter	magnetized iron calorimeter
fiducial mass	$22.5\,\mathrm{kt}\ (1\,000\,\mathrm{kt})$	$20\,\mathrm{kt}$	$10\,\mathrm{kt}\ (50\,\mathrm{kt})$
energy range	$0.4 - 1.2\,\mathrm{GeV}$	$1 - 5\,\mathrm{GeV}$	$4 - 50\,\mathrm{GeV}$
energy resolution	5%	10%	20%
signal efficiency	0.5	0.5	0.45
NC rejection	0.01	0.001	$< 10^{-5}$
CID	–	–	$< 10^{-5}$
backround uncertainty	5%	5%	5%

- **threshold effects** for the magnetized iron calorimeter
  linear rise of the **efficiency** between $4\,\mathrm{GeV}$ and $20\,\mathrm{GeV}$

- **liquid Argon TPC ?**

5 Considered Scenarios

	JHF-SK	NuMI	NuFact-I
detector	water cherenkov	low-Z calorimeter	10kt magnetized iron calorimter
baseline	$295\,\mathrm{km}$	$735\,\mathrm{km}$	$3\,000\,\mathrm{km}$
matter density	$2.8\,\mathrm{g\,cm^{-3}}$	$2.8\,\mathrm{g\,cm^{-3}}$	$3.5\,\mathrm{g\,cm^{-3}}$
L/E_{peak}	$378\,\mathrm{km\,GeV^{-1}}$	$337\,\mathrm{km\,GeV^{-1}}$	$90\,\mathrm{km\,GeV^{-1}}$

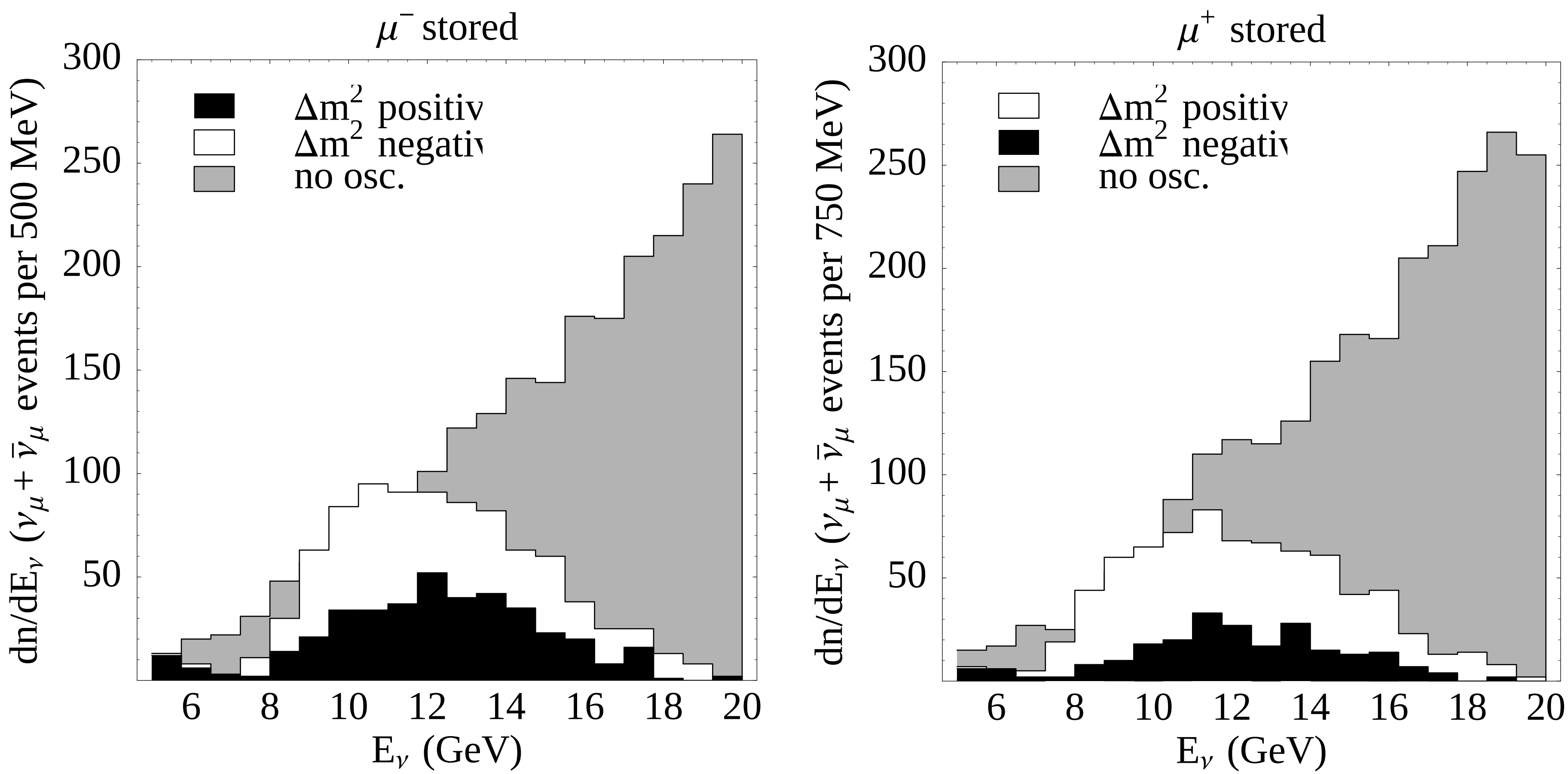
- **JHF-HK** $\Rightarrow$ $\simeq 95$ times more luminosity than JHF-SK plus anti-neutrino running
- **NuFact-II** $\Rightarrow$ $\simeq 42$ times more luminosity than NuFact-I

Itow et al., A. Para, A. Blondel et al.

	JHF-SK	NuMI	JHF-HK	NuFact-I	NuFact-II
signal	139.0	387.5	13 180.0	1 522.8	64 932.6
background	23.3	53.3	2 204.6	4.2	180.3
S/N	6	6	6	360	360

Disappearance Channels: Qualitative Picture

Large event rates $\Rightarrow$ spectral information $\Rightarrow$ NuFact example:



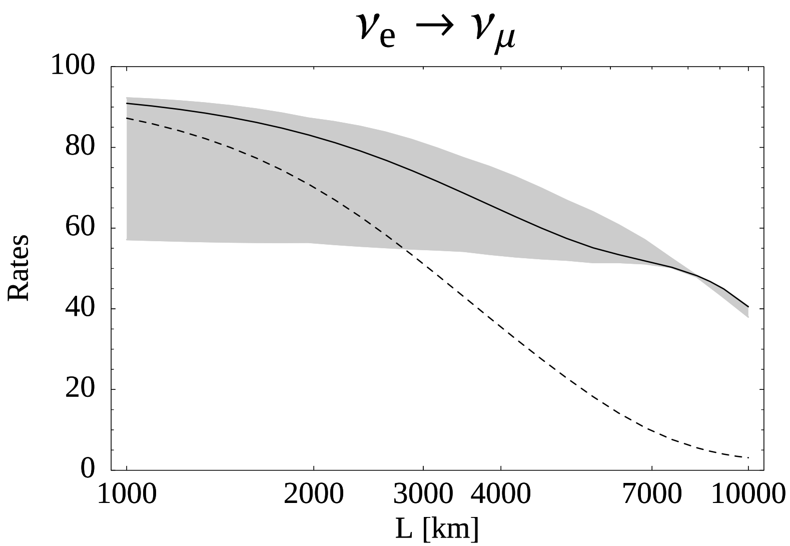
$\Rightarrow$ Δm^2_{31}, θ_{23} **and for not too small** θ_{13} **also** $sign(\Delta m^2_{31})$

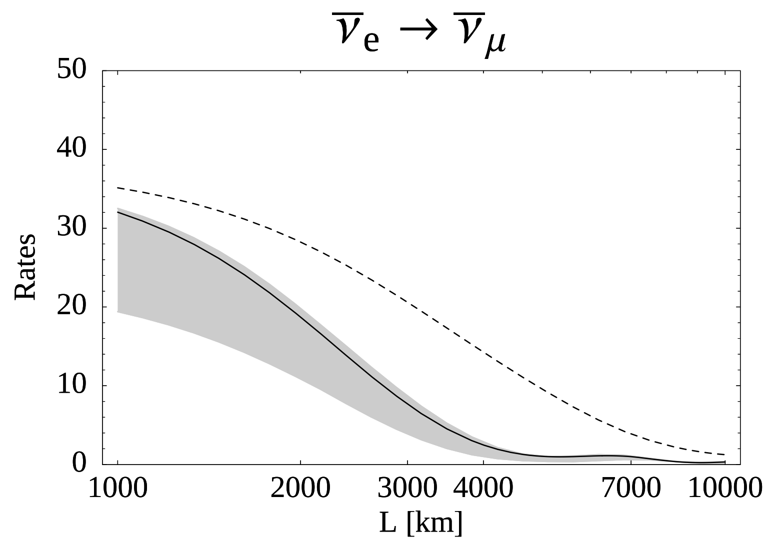
Cervera et al., Barger & Geer & Raja & Whisnant, DeRujula et al., Donini et al., Bueno & Campanelli & Rubbia, Albright et al.. Yasuda, Minakata, Shrock et al., Barenboim & De Gouvea & Szleper & Velasco, Freund & ML & Petcov & Romanino, Freund & Huber & ML, ...

Appearance Channels: Qualitative Picture

Small event rates $\Rightarrow$ total rates & little spectral information

NuFact:
$E_\mu = 50$ **GeV**
$\sin^2 2\theta_{13} = 0.01$
LMA





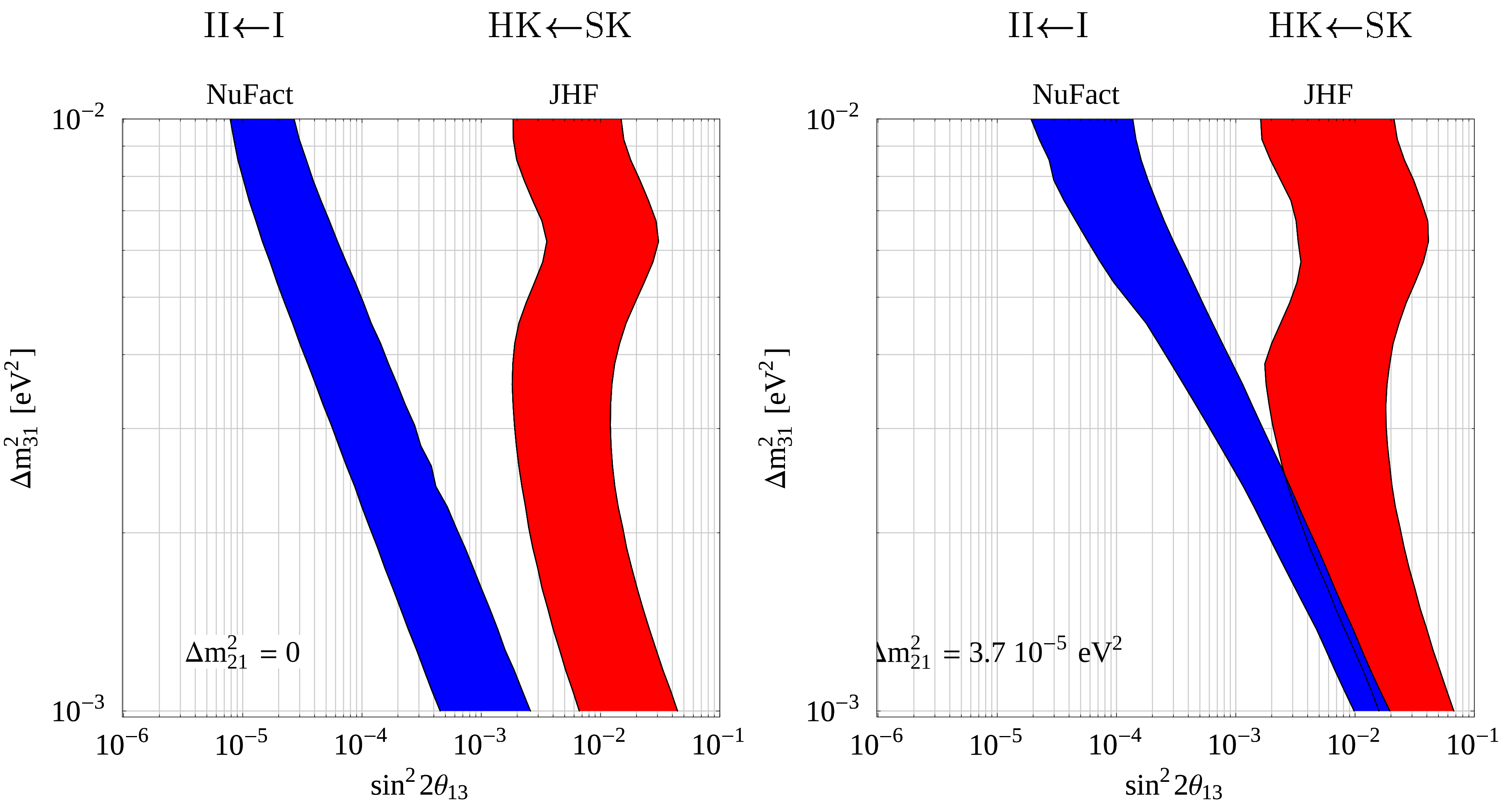
solid lines=matter ($\delta = 0$), dashed lines=vacuum ($\delta = 0$), grey band=all CP phases

Comparable matter and $\not{CP}$ effects $\Rightarrow$ separation
CP-phase $\Rightarrow$ short(er) baseline
matter effects, sign of δm^2 $\Rightarrow$ large L

For all solar solutions: θ_{13} and maybe $sign(\Delta m^2_{31})$
For LMA without external input: $\simeq \theta_{12}$, $\simeq \Delta m^2_{21}$
With external input for θ_{12} and Δm^2_{21}: $\Rightarrow$ δ_{CP}

Sub-Leading Parameters: $\sin^2 2\theta_{13}$ Sensitivity

All parameter correlations must be taken into account



- **sizable Δm^2_{31} and Δm^2_{21} dependence of sensitivity limits**
- **up to an order of magnitude difference**
- **effect strongest for short baselines**

Impact of Systematical Errors

Signal

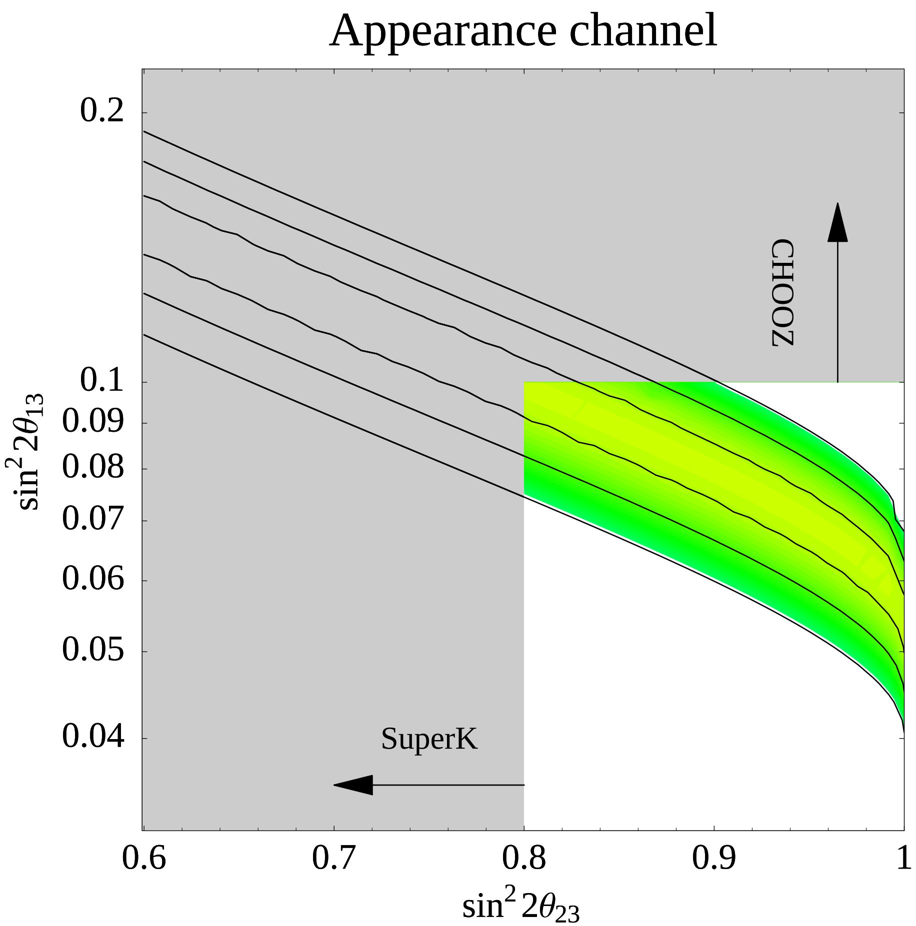
- use the **information in the energy spectrum**
- overall **normalization error** of 5% is acceptable
- **energy calibration error** of 5% is acceptable

Background

- **backgrounds limit sensitivity**
- **energy spectrum** helps to reduce the impact of the background
- background uncertainty becomes important when $\sigma_{b_n} \geq 1/\sqrt{b}$
  for $\sigma_{b_n} = 5\% \Leftrightarrow b = 400$

Background uncertainties may dominate high statistics experiments

Impact of Correlations



⇐ **Eample**

- **multi-parameter problem**
- **highly non-linear**
- **complex topology**

⇒ **use all available information:**

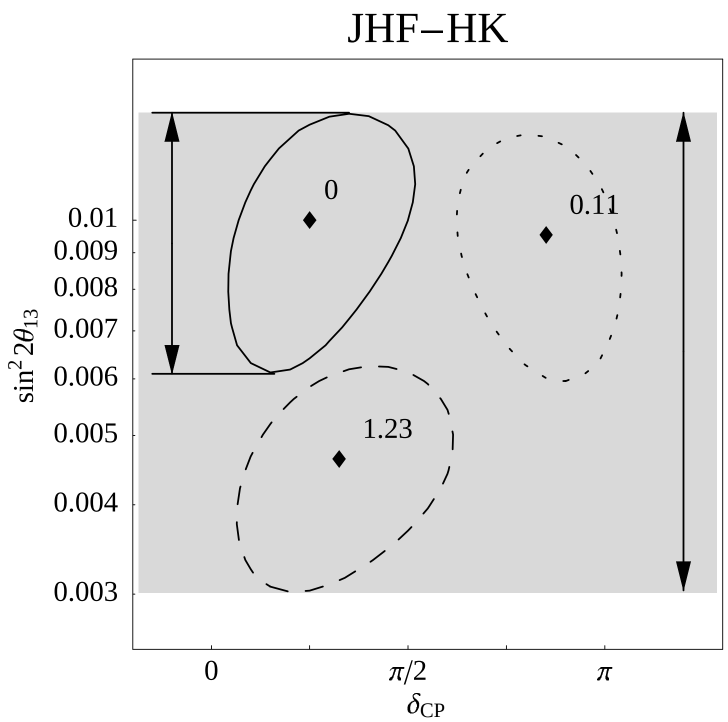
- **appearance & disappearance channel**
- **energy information**
- **external input**

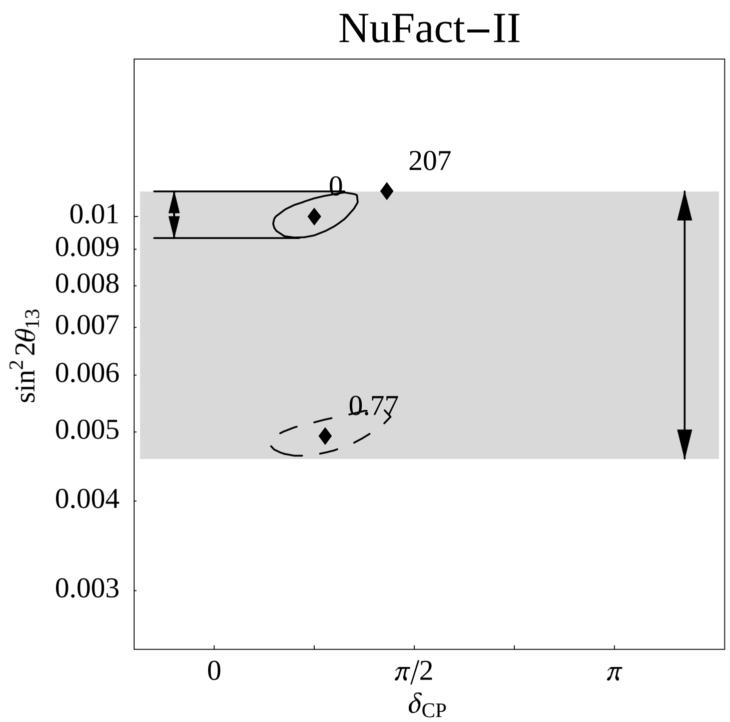
Impact of Degeneracies

Complex parameter dependence in appearance probabilities ⇒ multiple solutions

3 degeneracies:

- δ_{CP}-θ_{13}
- δ_{CP}-sgnΔm^2_{31}
- θ_{23}-$\pi/2 - \theta_{23}$





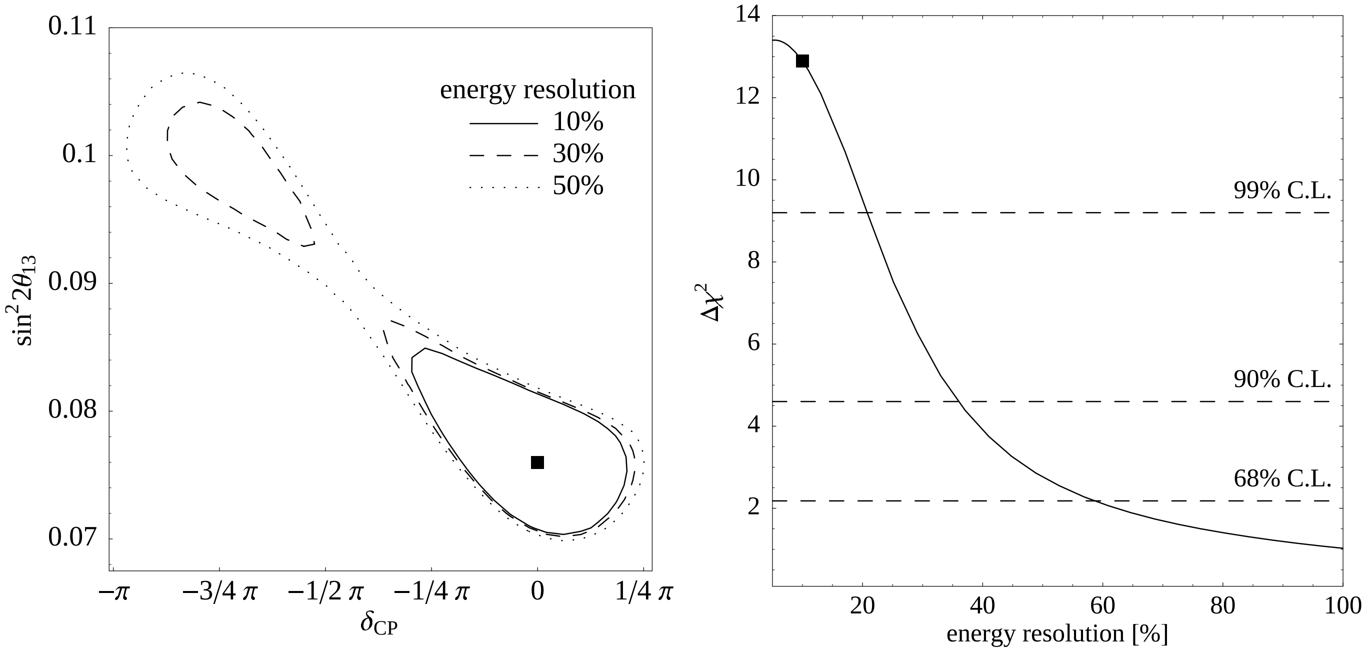
Full analysis:

- δ_{CP}-θ_{13} **is transformed to a correlation ⇔ neutrino beam only**
- δ_{CP}-sgnΔm^2_{31} **remains**
- θ_{23}-$\pi/2 - \theta_{23}$ **remains**
- **combined or seperate regions**

Cervera et al., Barger et al., Burguett-Castell et al., Minakata et al., Huber, ML, Winter

Medium Baseline & Good Resolution

Correlation of $\delta_{\rm CP}$ with θ_{13}



$$E = 50\,\mathrm{GeV},\; L = 3\,000\,\mathrm{km},\; \Delta m^2_{21} = 10^{-4}\,\mathrm{eV}^2$$

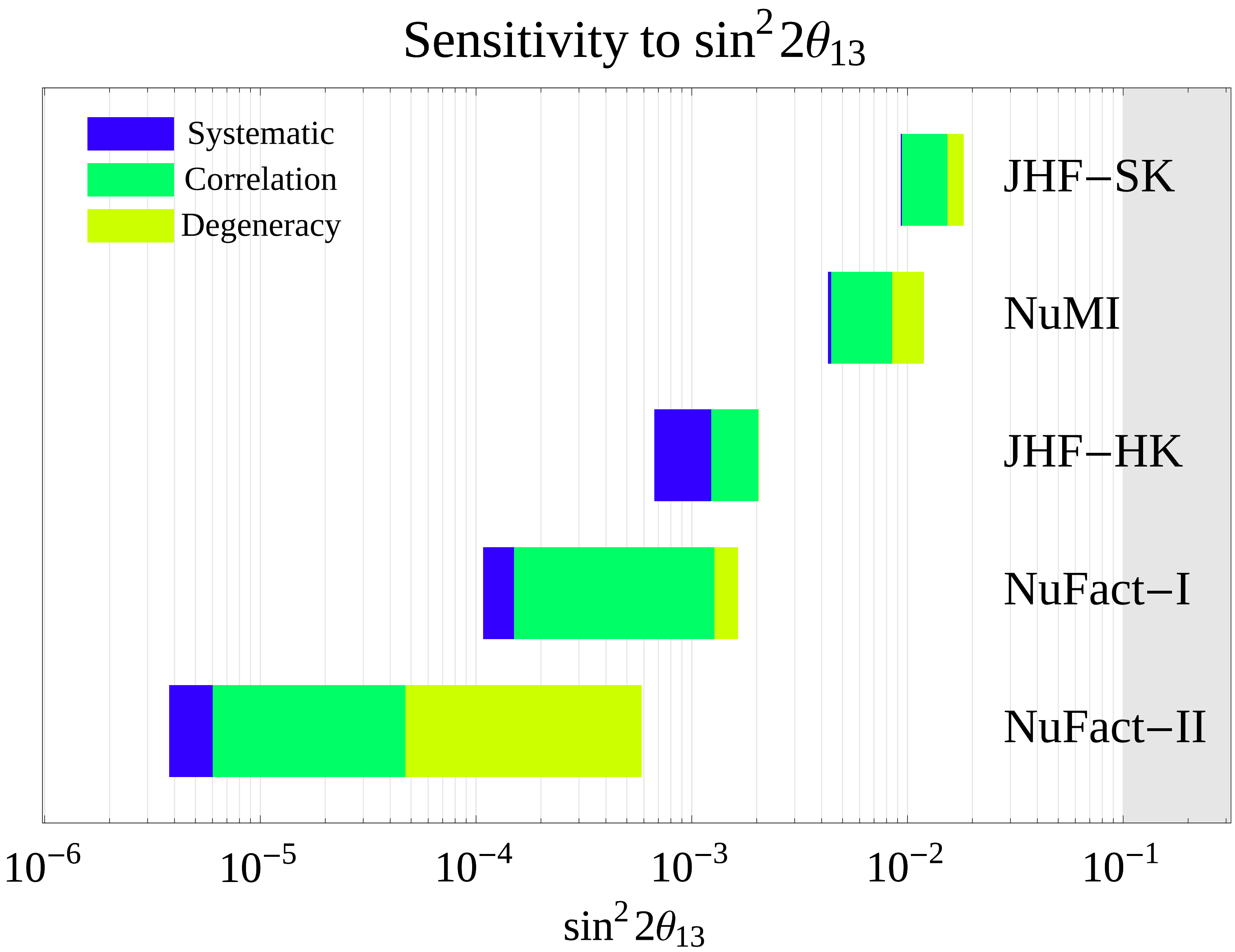
- two degenerate solutions
- depends strongly on energy resolution
- degeneracy lifted for energy resolution better than 25%

degeneracy can be resolved with spectral information $\Leftrightarrow$ resolution & flux

Burguet-Castell, Gavela, Gomez-Cadenas, Hernandez, Mena, Freund, Huber, ML

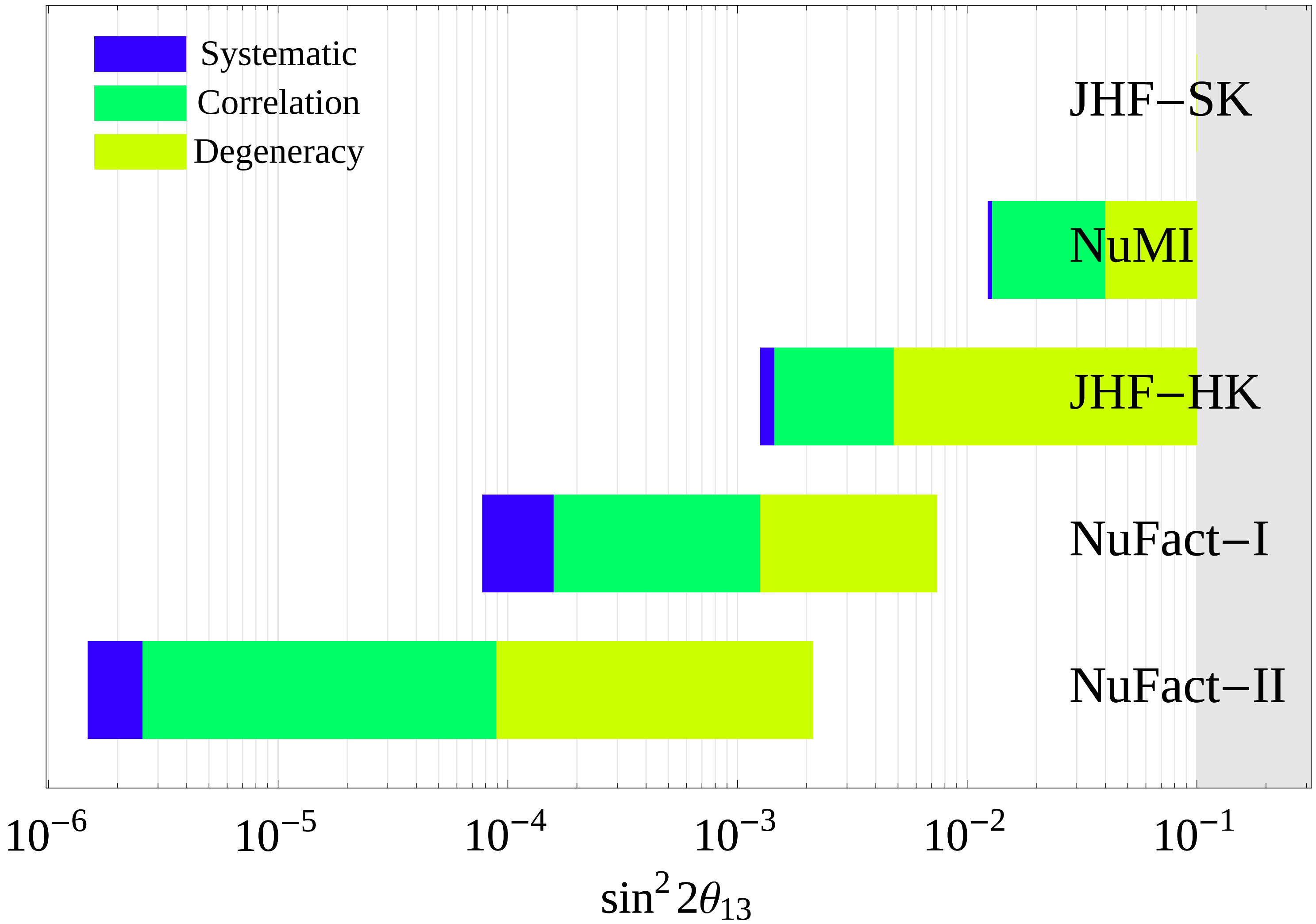
Comparative study including all effects

Huber, ML, Winter, hep-ph/0204352



- **Different sensitivity reductions by systematics**
- **Correlations & degeneracies lead to severe limitations**
- **Improvements by combining experiments**

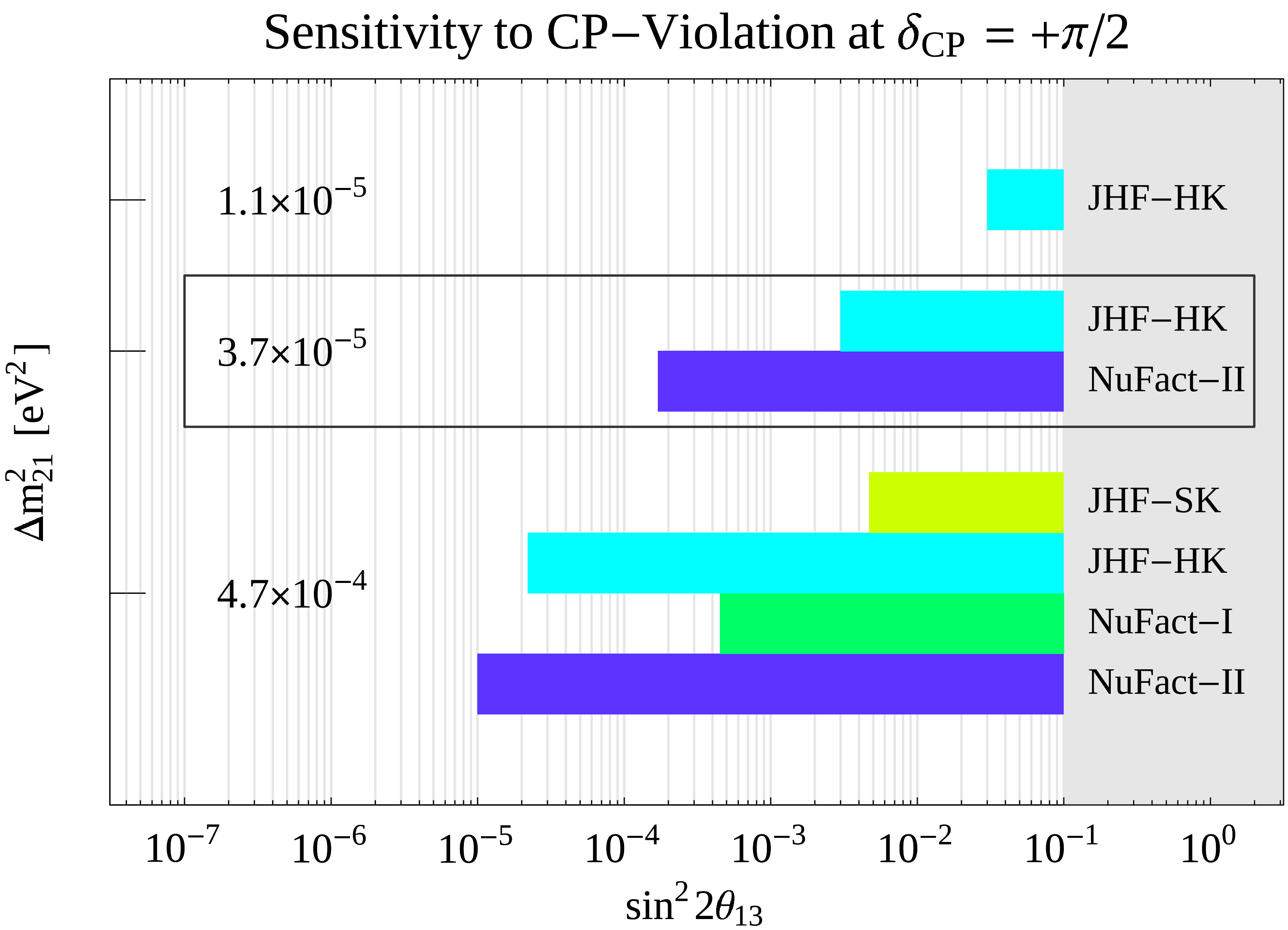
Sensitivity to the sign of Δm^2_{31}



- $sign(\Delta m^2_{31})$ **very hard to determine with superbeams**
- **degeneracies with δ_{CP} are the main problem**

$\Rightarrow$ **combine experiments!** **Huber, ML, Winter, hep-ph/0204352**

Measurements of CP-violation



- **CP violation** with high luminosity superbeams **feasable**
- **sensitivity is δ_{CP} dependent**

Huber, ML, Winter, hep-ph/0204352

T-Violating Effects

E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, NPB 608, 394 (2001), hep-ph/0105029.

CP, T and CPT Differences:

$$
\begin{aligned}
\Delta P^{CP}_{\alpha\beta} &\equiv P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \\
\Delta P^{T}_{\alpha\beta} &\equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) \\
\Delta P^{CPT}_{\alpha\beta} &\equiv P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)
\end{aligned}
$$

CPT is conserved in local QFT $\Rightarrow$ $\mathbf{P}(\nu_\alpha \to \nu_\beta) \equiv P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$

$$
\Delta P^{CP}_{\alpha\beta} \quad + \quad \Delta P^{T}_{\overline{\alpha\beta}} \quad = \quad \Delta P^{CPT}_{\alpha\beta} \quad = \quad 0
$$

CPT & vacuum $\Rightarrow$ correlated CP and T violation from NMS matrix

Oscillations in Matter: e^- and nucleons, i.e. particles, no antiparticles $\Rightarrow$

CP and CPT are violated by matter

$\Rightarrow$ Non-trivial interplay of fundamental and matter-induced T violation

Experimental Problem:

- No direct test of T-violation: Direction of time cannot be changed
- Alternative: ν-osc. forward in time with initial and final flavors interchanged

  $\Rightarrow$ Interchange of source and detector $\Leftrightarrow$ asymmetric density profile

T violation in neutrino oscillations:

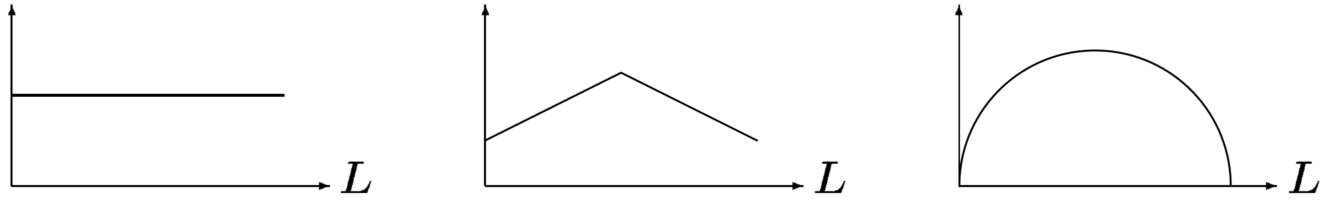
fundamental (intrinsic) T violation $\Leftrightarrow$ non-vanishing δ_{CP}
matter-induced T violation $\Leftrightarrow$ asymmetric profiles

$$\Delta P^T_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

In Vacuum: CPT $\Rightarrow$ **T violation** $\Leftrightarrow$ **CP violation**

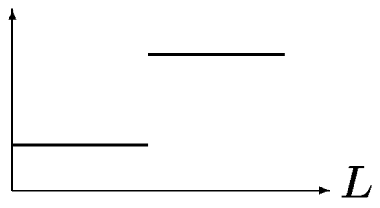
Matter: 2 ν-flavors $\Rightarrow$ **NO T-violating effects:** $P_{e\mu} = P_{\mu e}$ $\Rightarrow$ $\Delta P_{e\mu} = 0$

Matter: 3 ν-flavors, symmetric matter density profiles



$\Rightarrow$ **If $\delta_{CP} = 0$, then $\Delta P^T_{\alpha\beta} = 0$**

Matter: 3 ν-flavors, asymmetric matter density profiles



$\Rightarrow$ **General Case**

T-odd probability difference: Arbitrary density profile, small θ_{13} and δ/Δ

$$\begin{aligned}\Delta P^T_{e\mu} &\simeq -2s_{23}c_{23}\,\mathrm{Im}\,[\beta^*(A-C^*)] \\ &\simeq -2s_{13}s_{23}c_{23}\left(\Delta - s^2_{12}\delta\right)\mathrm{Im}\left[e^{-i\delta_{CP}}\beta^*\left(A_a - C^*_a\right)\right]\end{aligned}$$

where

$$s_{ij} \equiv \sin\theta_{ij}, \quad c_{ij} \equiv \cos\theta_{ij};^{1}, \quad \delta \equiv \frac{\Delta m^2_{21}}{2E_\nu}, \quad \Delta \equiv \frac{\Delta m^2_{31}}{2E_\nu};$$

$$A_a \equiv \alpha \int_{t_0}^{t} \alpha^* f\, dt' + \beta \int_{t_0}^{t} \beta^* f\, dt', \quad C_a \equiv f \int_{t_0}^{t} \alpha f^*\, dt'.$$

$\alpha = \alpha(t, t_0)$, $\beta = \beta(t, t_0)$ given by solutions of 2-flavor problem in the (1,2)-sector and $f = f(t, t_0) = \exp\left\{-i\int_{t_0}^{t}\left(\Delta - \frac{1}{2}[\delta + V(t')]\right)dt\right\}$

In addition: $\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$

$\Delta P^T_{e\mu}$ has been explicitely calculated for
1) matter consisting of two layers of constant density and
2) arbitrary matter density profile in the adiabatic approximation.

[1] $\theta_{12} = \theta_3, \theta_{13} = \theta_2, \theta_{23} = \theta_1$

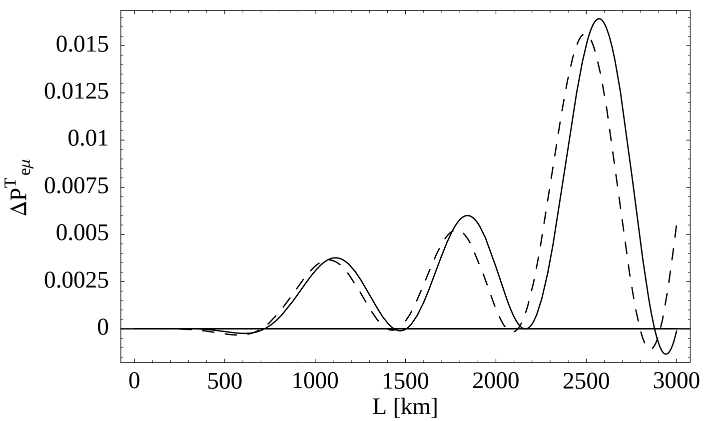
Low energy regime ($\delta = \Delta m^2_{21}/(2E_\nu) \gtrsim V_{1,2}$), two layers:

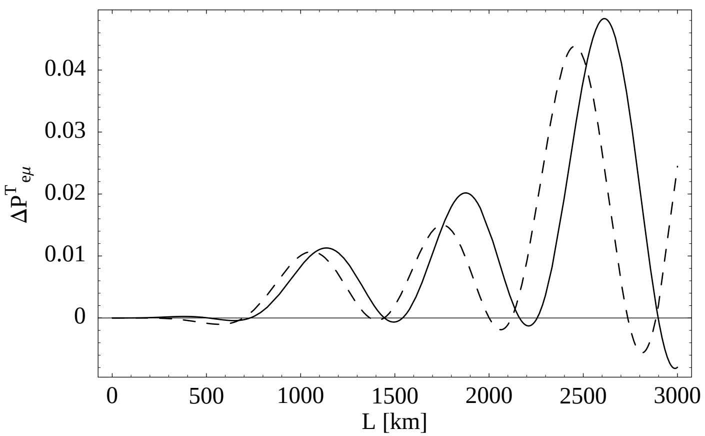
$$\Delta P^T_{\alpha\beta} \simeq \cos\delta_{CP} \cdot 8 \cdot \underbrace{s_{12}c_{12}s_{13}s_{23}c_{23}\frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}}_{=J_{\rm eff}} \times \{s_1 s_2 [Y - \cos(\Delta_1 L_1 + \Delta_2 L_2)]\}$$

$$+ \sin\delta_{CP} \cdot 4 s_{13}s_{23}c_{23}X_1 [Y - \cos(\Delta_1 L_1 + \Delta_2 L_2)]$$

$\cos\delta_{CP} \simeq$ **matter-induced T violation,** $\sin\delta_{CP} \simeq$ **fundamental T violation**

$\Delta P^T_{e\mu}(L)$**:**

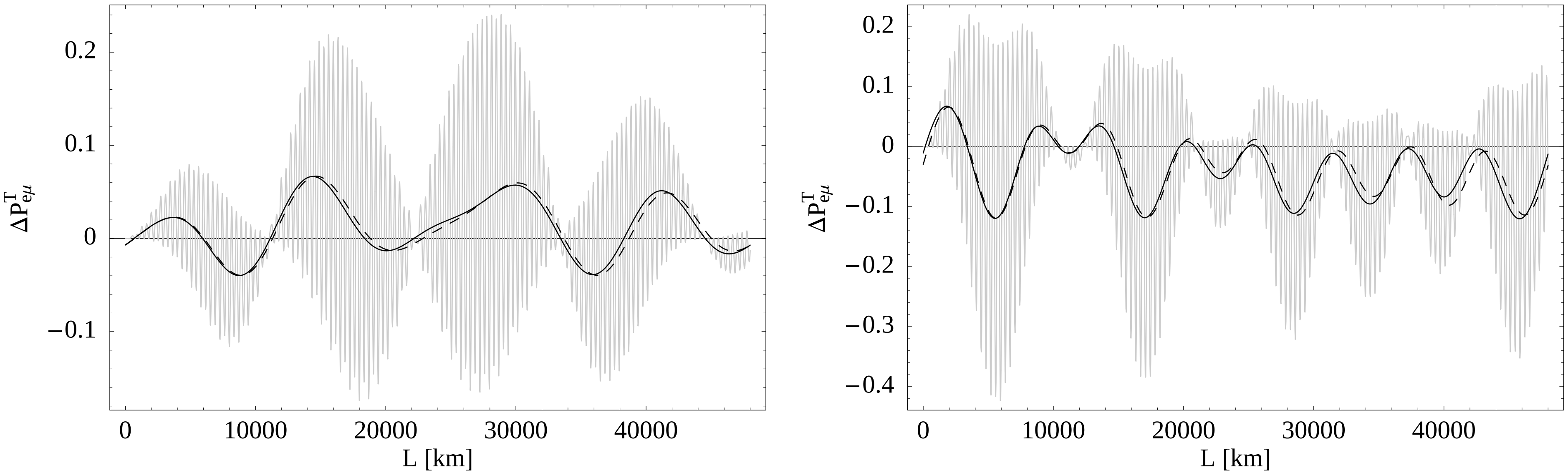




$L_1 = L_2 = L/2; \quad \rho_1 = 1\,\mathrm{g/cm^3},\ \rho_2 = 3\,\mathrm{g/cm^3},\ E_\nu = 1\ GeV$

solid curve - analytic result , dashed curve - numerical result

$\Delta P^T_{e\mu} = \Delta P^T_{e\mu}(L)$:



$$L_1 = L_2 = L/2; \quad \rho_1 = 0,\ \rho_2 = 6.4\,\mathrm{g/cm}^3$$

- **grey curves - analytic results**
- **black solid & dashed curves - averaged over fast oscillations**

$E_\nu = 0.5$ GeV $\Rightarrow$ **Large $\Delta m^2_{\mathrm{atm}} = \Delta m^2_{31}$ oscillations are very fast!**

Left plot: Parameters as in P.M. Fishbane and P. Kaus, PLB 506, 275 (2001)

Right plot: Larger values of θ_{13} and Δm^2_{21}.

NuFact on Earth $\Leftrightarrow$ $\simeq$ symmetric profiles $\Leftrightarrow$ theoretical error in studies

CPT-Violating Effects

S. Bilenky, M. Freund, M. Lindner, T. Ohlsson and W. Winter, Phys. Rev. D 65 (2002) 073024

CPT-violation

$\Downarrow$

Physics beyond local QFT

$\Downarrow$

“Planck Scale Physics”, “extra dimensions”, ...

- **CPT violation was used to accommodate LSND result**
- **Here: Precision measurements $\Rightarrow$ interesting limits**

Sensitivity to small CPT-Violating Effects at NuFact

CPT-invariance ⇒ $\mathrm{P}(\nu_\alpha \rightarrow \nu_{\alpha'}) = \mathrm{P}(\overline{\nu_{\alpha'}} \rightarrow \overline{\nu_\alpha})$

CPT-violation ⇒ **Masses and mixings differ for neutrinos and antineutrinos.**
Neutrinos: m_i, U **anti-neutrinos:** $\overline{m}_i$, $\overline{U}$
Consequence: $\mathrm{P}(\nu_\alpha \rightarrow \nu_{\alpha'}) \neq \mathrm{P}(\overline{\nu_{\alpha'}} \rightarrow \overline{\nu_\alpha})$

Limit CPT-violation in the disappearance channels at NuFact

- high event rates
- no beam contamination
- small matter effects
- large oscillation effects

⇒ **Exclusion limits for tiny CPT-violating effects**

Consider ν- and $\overline{\nu}$ channels as independent experiments:

$$\delta \equiv |\Delta m^2_{32} - \Delta \overline{m}^2_{32}| \ , \quad \epsilon \equiv |\sin^2 2\theta_{23} - \sin^2 2\overline{\theta}_{23}|$$

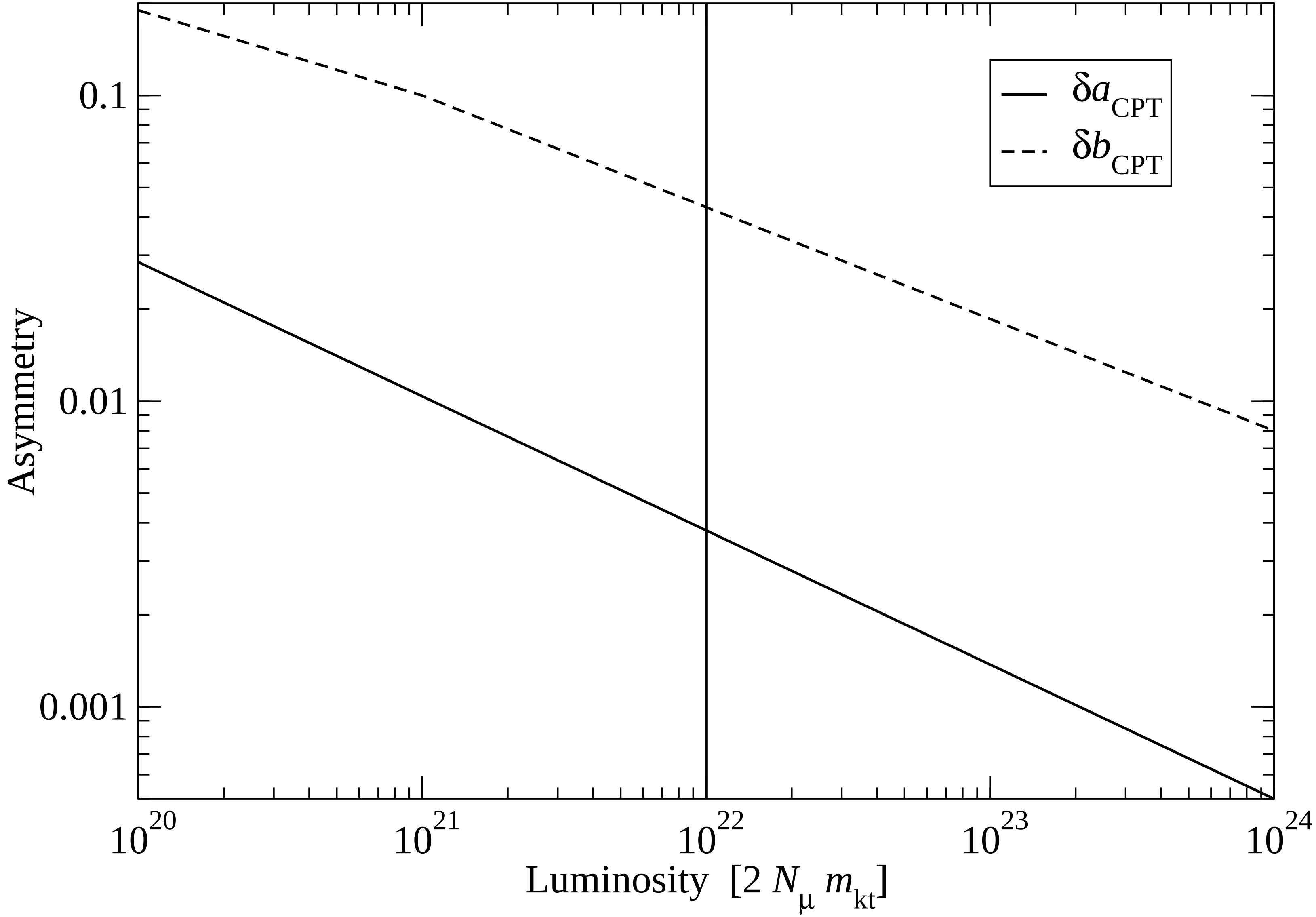
For $m_1 \ll \sqrt{\Delta m^2_{sun}}$ **(hierarchical) and** $|m_3 - \overline{m}_3| \ll (m_3)_{\text{average}}$

$$\delta \simeq 2\, a_{\text{CPT}}\, \Delta m^2_{32} \qquad \textbf{with} \qquad a_{\text{CPT}} \equiv \frac{|m_3 - \overline{m}_3|}{(m_3)_{\text{average}}}$$

$$\epsilon \simeq 2\, b_{\text{CPT}} \sqrt{\sin^2 2\theta_{23}} \sqrt{1 - \sin^2 2\theta_{23}}\ \arcsin \sqrt{\sin^2 2\theta_{23}} \qquad \textbf{with} \qquad b_{\text{CPT}} \equiv \frac{|\theta_{23} - \overline{\theta}_{23}|}{(\theta_{23})_{\text{average}}}$$

- **Sensitivity to possible CPT violation** $\Leftrightarrow$ **accuracy of** a_{CPT} **and/or** b_{CPT}
- **Compare** δ **and** ϵ **with respective relative statistical error** $\delta \Delta m^2_{32}$ **or** $\delta\theta_{23}$
- **Sensitivities** δa_{CPT} **and** δb_{CPT} **for** a_{CPT} **and** b_{CPT} **are given by:**

$$\delta a_{\text{CPT}} \sim \frac{\delta \Delta m^2_{32}}{2}$$
$$\delta b_{\text{CPT}} \sim \delta\theta_{23}$$



$E_\nu = 50\text{GeV}$, $L = 7000km$ (θ_{23}) / $3000km$ (Δm^2_{23}), 10^{20} **muons/y, 5 years, 10kt**

$a_{CPT} \leq 3.8 \cdot 10^{-3}$, $b_{CPT} \leq 4.3 \cdot 10^{-2}$ $\Leftrightarrow$ $|m_3 - \overline{m}_3| = 1.9 \cdot 10^{-4}$ eV

Conclusions

CP, T and CPT violation in neutrino oscillations offer exciting perspectives!

CP violation:

- intrinsic CP violation from MNS matrix
- extra CP violation from matter effects
- both can be measured in future LBL experiments
- separation strategies ...

T violation:

- in vacuum correlated to CP violation
- complex interplay between fundamental and matter effects
- very nice theoretical features
- for LBL on earth small $\Leftrightarrow$ theoretical precision limit

CPT violation:

- in local QFT not allowed
- matter induced CPT violation
- fundamental CPT violation $\Leftrightarrow$ Planck scale phyics